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Cooperative tasking for multi-agent systems

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Abstract

New advances in large scale distributed systems have amazingly offered complex functionalities through parallelism of simple and rudimentary components. The key issue in cooperative control of multi-agent systems is the synthesis of local control and interaction rules among the agents such that the entire controlled system achieves a desired global behavior. For this purpose, three fundamental problems have to be addressed: (1) task decomposition for top-down design, such that the fulfillment of local tasks guarantees the satisfaction of the global task, by the team; (2) fault-tolerant top-down design, such that the global task remain decomposable and achievable, in spite of some failures, and (3) design of interactions among agents to make an undecomposable task decomposable and achievable in a top-down framework. The first two problems have been addressed in our previous works, by identifying necessary and sufficient conditions for task automaton decomposition, and fault-tolerant task decomposability, based on decision making on the orders and selections of transitions, interleaving of synchronized strings and determinism of bisimulation quotient of local task automata. This paper deals with the third problem and proposes a procedure to redistribute the events among agents in order to enforce decomposability of an undecomposable task automaton. The decomposability conditions are used to identify the root causes of undecomposability which are found to be due to over-communications that have to be deleted, while respecting the fault-tolerant decomposability conditions; or because of the lack of communications that require new sharing of events, while considering new violations of decomposability conditions. This result provides a sufficient condition to make any undecomposable deterministic task automaton decomposable in order to facilitate cooperative tasking. Illustrative examples are presented to show the concept of task automaton decomposabilization.

I. INTRODUCTION

With new advances in technology and emergence of large scale complex systems [1], [2], there is an ever-increasing demand for cooperative control of distributed systems with sophisticated specifications [3], [4], [5], [6] which impose new challenges that fall beyond the traditional methods [7], [8], [9], [5]. Conventional approaches either consider the team of agents as a monolithic plant to be controlled by a centralized unit, or design and iteratively adjust local controllers, in a bottom-up structure, to generate a behavior closed to a desired global behavior. Although the latter approach offers more flexibility, scalability and functionality with lower cost, due to local actuation and communications of agents [10], [11], [12], they fail to guarantee a given global specification [13]. For this purpose, top-down cooperative control aims at formal design of local controllers in order to collectively achieve the global specification, by design [14], [15].

To address the top-down cooperative control, three fundamental questions are evoked: The first question is the task decomposition problem that is interested in understanding of whether all tasks are decomposable, and if not, what are the conditions for task decomposability. It furthermore asks that if the task is decomposable and local controllers are designed to satisfy local tasks, whether the whole closed loop system satisfies the global specification. Subsequently, the second question refers to the cooperative control under event failures, and would like to know if after the task decomposition and local controller designs for global satisfaction, some events fail in some agents, then whether the task still remains decomposable and globally satisfied, in spite of event failures. As another follow-up direction, the third question investigates the way to make an undecomposable task decomposable through modification of local agents in order to accomplish the proposed cooperative control.

For cooperative control of logical behaviors [16], represented in automata [17], [18], the first question (task decomposability for cooperative tasking) was addressed in our previous work [19], by decomposing a given global task automaton into two local task automata such that their parallel composition bisimulates the original task automaton. By using the notion of shared events, instead of common events and incorporating the concept of global decision making on the orders and selections between the transitions, the decomposability result was generalized in [20] to an arbitrary finite number of agents. Given a deterministic task automaton, and a set of local

event sets, necessary and sufficient conditions were identified for task automaton decomposability based on decision making on the orders and selections of transitions, interleaving of synchronized strings and determinism of bisimulation quotient of local automata. It was also proven that the fulfillment of local task automata guarantees the satisfaction of the global specification, by design.

The second question, cooperative tasking under event failure, was investigated in [21], by introducing a notion of passive events to transform the fault-tolerant task decomposability problem to the standard automaton decomposability problem in [20]. The passivity was found to reflect the redundancy of communication links, based on which the necessary and sufficient conditions have been then introduced under which a previously decomposable task automaton remains decomposable and achievable, in spite of events failures. The conditions ensure that after passive failures, the team of agents maintains its capability for global decision making on the orders and selections between transitions; no illegal behavior is allowed by the team (no new string emerges in the interleavings of local strings) and no legal behavior is disabled by the team (any string in the global task automaton appears in the parallel composition of local automata). These conditions interestingly guarantee the team of agents to still satisfy its global specification, even if some local agents fail to maintain their local specifications.

This paper deals with the third question to investigate how to make undecomposable task automata decomposable in order for cooperative tasking of multi-agent systems. For a global task automaton that is not decomposable with respect to given local event sets, the problem is particularly interested in finding a way to modify the local task automata such that their parallel composition bisimulates the original global task automaton, to guarantee its satisfaction by fulfilling the local task automata.

Decomposition of different formalisms of logical specification have been reported in the literature. Examples of such methods can be seen for decomposition of a specification given in CSP [22], decomposition of a LOTOS [23], [24], [25] and decomposition of petri nets [26], [27]. The problem of automaton decomposabilization has been also studies in computer science literature. For example, [28] characterized the conditions for decomposition of asynchronous automata in the sense of isomorphism based on the maximal cliques of the dependency graph. The isomorphism equivalence used in [28] is however a strong condition, in the sense that two isomorphic automata are bisimilar but not vise versa [17]. Moreover, [28] considers a set of events to be attributed to a number of agents, with no predefinition of local event sets.

While event attribution is suitable for parallel computing and synthesis problems in computer science, control applications typically deal with parallel distributed plants [29] whose events are predefined by the set of sensors, actuators and communication links across the agents. Therefore, it would be advantageous to find a way to make an undecomposable automaton decomposable with respect to predefined local event sets, by modifying local task automata. Since the global task automaton is fixed, one way to modify the local task automata is through the modification in local event sets, which is the main theme of this paper. Another related work is [30] that proposes a method for automaton decomposabilization by adding synchronization events such that the parallel composition of local automata is observably bisimilar to the original automaton. The approach in [30], however, allows to add synchronization events to the event set that will enlarge the size of global event set. Our work deals with those applications with fixed global event sets and predefined distribution of events among local agents, where enforcing the decomposability is not allowed by adding the new synchronization events, but instead by redistribution of the existing events among the agents.

For this purpose, we propose an algorithm that uses previous results on task decomposition [19], [20] to identify and overcome dissatisfaction of each decomposability condition. The algorithm first removes all redundant communication links using the fault-tolerant result [21]. As a result, any violation of decomposability conditions, remained after this stage, is not due to redundant communication links, and hence cannot be removed by means of link deletions. Instead, the algorithm proceeds by establishing new communication links to provide enough information to facilitate the task automaton decomposition. Since each new communication link may overcome several violations of decomposability conditions, the algorithm may offer different options for link addition, leading to the question of optimal decomposability with minimum number of communication links. It is found that if link additions impose no new violations of decomposability conditions, then it is possible to make the automaton decomposable with minimum number of links. However, it is furthermore shown that, in general, addition of new communication links may introduce new violations of decomposability conditions that in turn require establishing new communication links. In such cases, the optimal path depends on the structure of the automaton and requires a dynamic exhaustive search to find the sequence of link additions with minimum number of links. Therefore, in case of new violations, a simple sufficient condition is proposed to provide a feasible suboptimal solution to enforce the decomposability,

without checking of decomposability conditions after each link addition. This approach can decompose any deterministic task automaton, after which, according to the previous results, designing local controllers such that local specification are satisfied, guarantees the fulfillment of the global specification, by design.

The rest of the paper is organized as follows. Preliminary lemmas, notations, definitions and problem formulation are represented in Section II. This section also establishes the links to previous works on task automaton decomposition and fault-tolerant decomposition results. Section III proposes an algorithm to make any undecomposable deterministic automaton decomposable by modifying its local event sets. Illustrative examples are also given to elaborate the concept of task automaton decomposabilization. Finally, the paper concludes with remarks and discussions in Section IV. Proofs of the lemmas are readily given in the Appendix.

II. PROBLEM FORMULATION

A. Definitions and notations

We first recall the definitions and notations used in this paper.

A *deterministic automaton* is a tuple $A := (Q, q_0, E, \delta)$ consisting of a set of states Q ; an initial state $q_0 \in Q$; a set of events E that causes transitions between the states, and a transition relation $\delta \subseteq Q \times E \times Q$, with partial map $\delta : Q \times E \rightarrow Q$, such that $(q, e, q') \in \delta$ if and only if state q is transited to state q' by event e , denoted by $q \xrightarrow{e} q'$ (or $\delta(q, e) = q'$). A *nondeterministic automaton* is a tuple $A := (Q, q_0, E, \delta)$ with a partial transition map $\delta : Q \times E \rightarrow 2^Q$, and if hidden transitions (ε -moves) are also possible, then a nondeterministic automaton with hidden moves is defined as $A := (Q, q_0, E \cup \{\varepsilon\}, \delta)$ with a partial map $\delta : Q \times (E \cup \{\varepsilon\}) \rightarrow 2^Q$. For a nondeterministic automaton the initial state can be generally from a set $Q_0 \subseteq Q$. Given a nondeterministic automaton A , with hidden moves, the ε -closure of $q \in Q$, denoted by $\varepsilon_A^*(q) \subseteq Q$, is recursively defined as: $q \in \varepsilon_A^*(q)$; $q' \in \varepsilon_A^*(q) \Rightarrow \delta(q', \varepsilon) \subseteq \varepsilon_A^*(q)$. The transition relation can be extended to a finite string of events, $s \in E^*$, where E^* stands for *Kleene-Closure* of E (the set of all finite strings over elements of E). For an automaton without hidden moves, $\varepsilon_A^*(q) = \{q\}$, and the transition on string is inductively defined as $\delta(q, \varepsilon) = q$ (empty move or silent transition), and $\delta(q, se) = \delta(\delta(q, s), e)$ for $s \in E^*$ and $e \in E$. For an automaton A , with hidden moves, the extension of transition relation on string, denoted by

$\delta : Q \times E^* \rightarrow 2^Q$, is inductively defined as: $\forall q \in Q, s \in E^*, e \in E: \delta(q, \varepsilon) := \varepsilon_A^*(q)$ and $\delta(q, se) = \varepsilon_A^*(\delta(\delta(q, s), e)) = \bigcup_{q' \in \delta(q, s)} \left\{ \bigcup_{q'' \in \delta(q', e)} \varepsilon_A^*(q'') \right\}$ [18].

The operator $Ac(\cdot)$ [17] is then defined by excluding the states and their attached transitions that are not reachable from the initial state as $Ac(A) = (Q_{ac}, q_0, E, \delta_{ac})$ with $Q_{ac} = \{q \in Q \mid \exists s \in E^*, q \in \delta(q_0, s)\}$ and $\delta_{ac} = \delta|_{Q_{ac} \times E \rightarrow Q_{ac}}$, restricting δ to the smaller domain of Q_{ac} . Since $Ac(\cdot)$ has no effect on the behavior of the automaton, from now on we take $A = Ac(A)$.

We focus on deterministic global task automata that are simpler to be characterized, and cover a wide class of specifications. The qualitative behavior of a deterministic system is described by the set of all possible sequences of events starting from the initial state. Each such a sequence is called a string, and the collection of strings represents the language generated by the automaton, denoted by $L(A)$. The existence of a transition over a string $s \in E^*$ from a state $q \in Q$ is denoted by $\delta(q, s)!$. Considering a language L , by $\delta(q, L)!$ we mean that $\forall \omega \in L : \delta(q, \omega)!$. For $e \in E, s \in E^*, e \in s$ means that $\exists t_1, t_2 \in E^*$ such that $s = t_1 e t_2$. In this sense, the intersection of two strings $s_1, s_2 \in E^*$ is defined as $s_1 \cap s_2 = \{e \mid e \in s_1 \wedge e \in s_2\}$. Likewise, $s_1 \setminus s_2$ is defined as $s_1 \setminus s_2 = \{e \mid e \in s_1, e \notin s_2\}$. For $s_1, s_2 \in E^*$, s_1 is called a sub-string of s_2 , denoted by $s_1 \leq s_2$, when $\exists t \in E^*, s_2 = s_1 t$. Two events e_1 and e_2 are called successive events if $\exists q \in Q : \delta(q, e_1)! \wedge \delta(\delta(q, e_1), e_2)!$ or $\delta(q, e_2)! \wedge \delta(\delta(q, e_2), e_1)!$. Two events e_1 and e_2 are called adjacent events if $\exists q \in Q : \delta(q, e_1)! \wedge \delta(q, e_2)!$.

To compare the task automaton and its decomposed automata, we use the *bisimulation relations*. Consider two automata $A_i = (Q_i, q_i^0, E, \delta_i)$, $i = 1, 2$. A relation $R \subseteq Q_1 \times Q_2$ is said to be a simulation relation from A_1 to A_2 if $(q_1^0, q_2^0) \in R$, and $\forall (q_1, q_2) \in R, \delta_1(q_1, e) = q'_1$, then $\exists q'_2 \in Q_2$ such that $\delta_2(q_2, e) = q'_2, (q'_1, q'_2) \in R$. If R is defined for all states and all events in A_1 , then A_1 is said to be similar to A_2 (or A_2 simulates A_1), denoted by $A_1 \prec A_2$ [17]. If $A_1 \prec A_2$, $A_2 \prec A_1$, with a symmetric relation, then A_1 and A_2 are said to be bisimilar (bisimulate each other), denoted by $A_1 \cong A_2$ [31]. In general, bisimilarity implies languages equivalence but the converse does not necessarily hold [32].

In these works *natural projection* is used to obtain local tasks, as local perspective of agents from the global task. Consider a global event set E and its local event sets $E_i, i = 1, 2, \dots, n$, with $E = \bigcup_{i=1}^n E_i$. Then, the natural projection $p_i : E^* \rightarrow E_i^*$ is inductively defined as $p_i(\varepsilon) = \varepsilon$,

and $\forall s \in E^*, e \in E : p_i(se) = \begin{cases} p_i(s)e & \text{if } e \in E_i; \\ p_i(s) & \text{otherwise.} \end{cases}$ Accordingly, inverse natural projection $p_i^{-1} : E_i^* \rightarrow 2^{E^*}$ is defined on an string $t \in E_i^*$ as $p_i^{-1}(t) := \{s \in E^* | p_i(s) = t\}$.

The natural projection is also defined on automata as $P_i : A \rightarrow A$, where, A is the set of finite automata and $P_i(A_S)$ are obtained from A_S by replacing its events that belong to $E \setminus E_i$ by ε -moves, and then, merging the ε -related states. The ε -related states form equivalent classes defined as follows. Consider an automaton $A_S = (Q, q_0, E, \delta)$ and a local event set $E_i \subseteq E$. Then, the relation \sim_{E_i} is the equivalence relation on the set Q of states such that $\delta(q, e) = q' \wedge e \notin E_i \Rightarrow q \sim_{E_i} q'$, and $[q]_{E_i}$ denotes the equivalence class of q defined on \sim_{E_i} . The set of equivalent classes of states over \sim_{E_i} , is denoted by Q/\sim_{E_i} and defined as $Q/\sim_{E_i} = \{[q]_{E_i} | q \in Q\}$ [28]. The natural projection of A_S into E_i is then formally defined as $P_i(A_S) = (Q_i = Q/\sim_{E_i}, [q_0]_{E_i}, E_i, \delta_i)$, with $\delta_i([q]_{E_i}, e) = [q']_{E_i}$ if there exist states q_1 and q'_1 such that $q_1 \sim_{E_i} q$, $q'_1 \sim_{E_i} q'$, and $\delta(q_1, e) = q'_1$.

To investigate the interactions of transitions between automata, particularly between $P_i(A_S)$, $i = 1, \dots, n$, the *synchronized product of languages* is defined as follows. Consider a global event set E and local event sets E_i , $i = 1, \dots, n$, such that $E = \bigcup_{i=1}^n E_i$. For a finite set of languages $\{L_i \subseteq E_i^* | i=1, \dots, n\}$, the synchronized product (language product) of $\{L_i\}$, denoted by $\bigcap_{i=1}^n L_i$, is defined as $\bigcap_{i=1}^n L_i = \{s \in E^* | \forall i \in \{1, \dots, n\} : p_i(s) \in L_i\} = \bigcap_{i=1}^n p_i^{-1}(L_i)$ [14].

Then, *parallel composition (synchronized product)* is used to define the composition of local task automata to retrieve the global task automaton, and to model each local closed loop system by compositions of its local plant and local controller automata. Let $A_i = (Q_i, q_i^0, E_i, \delta_i)$, $i = 1, 2$ be automata. The parallel composition (synchronous composition) of A_1 and A_2 is the automaton $A_1 || A_2 = (Q = Q_1 \times Q_2, q_0 = (q_1^0, q_2^0), E = E_1 \cup E_2, \delta)$, with δ defined as $\forall (q_1, q_2) \in Q, e \in E$:

$$\delta((q_1, q_2), e) = \begin{cases} (\delta_1(q_1, e), \delta_2(q_2, e)), & \text{if } \begin{cases} \delta_1(q_1, e)!, \delta_2(q_2, e)! \\ e \in E_1 \cap E_2 \end{cases}; \\ (\delta_1(q_1, e), q_2), & \text{if } \delta_1(q_1, e)!, e \in E_1 \setminus E_2; \\ (q_1, \delta_2(q_2, e)), & \text{if } \delta_2(q_2, e)!, e \in E_2 \setminus E_1; \\ \text{undefined,} & \text{otherwise.} \end{cases}$$

The parallel composition of A_i , $i = 1, 2, \dots, n$ is called *parallel distributed system* (or concurrent system), and is defined based on the associativity property of parallel composition [17] as

$$\prod_{i=1}^n A_i = A_1 \parallel \dots \parallel A_n = A_n \parallel (A_{n-1} \parallel (\dots \parallel (A_2 \parallel A_1))).$$

The set of labels of local event sets containing an event e is called *the set of locations* of e , denoted by $loc(e)$ and is defined as $loc(e) = \{i \in \{1, \dots, n\} | e \in E_i\}$.

Based on these definitions, a task automaton A_S with event set E and local event sets E_i , $i = 1, \dots, n$, $E = \bigcup_{i=1}^n E_i$, is said to be decomposable with respect to parallel composition and natural projections P_i , $i = 1, \dots, n$, when $\prod_{i=1}^n P_i(A_S) \cong A_S$.

B. Problem formulation

In [19], we have shown that not all automata are decomposable with respect to parallel composition and natural projections, and subsequently necessary and sufficient conditions were proposed for decomposability of a task automaton with respect to parallel composition and natural projections into two local event sets. These necessary and sufficient conditions were then generalized to an arbitrary finite number of agents, in [20], as

Lemma 1: (Corollary 1 in [20]) A deterministic automaton $A_S = (Q, q_0, E = \bigcup_{i=1}^n E_i, \delta)$ is decomposable with respect to parallel composition and natural projections P_i , $i = 1, \dots, n$ such that $A_S \cong \prod_{i=1}^n P_i(A_S)$ if and only if A_S satisfies the following decomposability conditions (DC):

- DC1: $\forall e_1, e_2 \in E, q \in Q: [\delta(q, e_1)! \wedge \delta(q, e_2)!]$
 $\Rightarrow [\exists E_i \in \{E_1, \dots, E_n\}, \{e_1, e_2\} \subseteq E_i] \vee [\delta(q, e_1 e_2)! \wedge \delta(q, e_2 e_1)!];$
- DC2: $\forall e_1, e_2 \in E, q \in Q, s \in E^*: [\delta(q, e_1 e_2 s)! \vee \delta(q, e_2 e_1 s)!]$
 $\Rightarrow [\exists E_i \in \{E_1, \dots, E_n\}, \{e_1, e_2\} \subseteq E_i] \vee [\delta(q, e_1 e_2 s)! \wedge \delta(q, e_2 e_1 s)!];$
- DC3: $\delta(q_0, \prod_{i=1}^n p_i(s_i))!, \forall \{s_1, \dots, s_n\} \in \tilde{L}(A_S), \exists s_i, s_j \in \{s_1, \dots, s_n\}, s_i \neq s_j$, where, $\tilde{L}(A_S) \subseteq L(A_S)$ is the largest subset of $L(A_S)$ such that $\forall s \in \tilde{L}(A_S) \exists s' \in \tilde{L}(A_S), \exists E_i, E_j \in \{E_1, \dots, E_n\}, i \neq j, p_{E_i \cap E_j}(s)$ and $p_{E_i \cap E_j}(s')$ start with the same event, and
- DC4: $\forall i \in \{1, \dots, n\}, x, x_1, x_2 \in Q_i, x_1 \neq x_2, e \in E_i, t \in E_i^*, \delta_i(x, e) = x_1, \delta_i(x, e) = x_2:$
 $\delta_i(x_1, t)! \Leftrightarrow \delta_i(x_2, t)!.$

The first two decomposability conditions require the team to be capable of decision on choice/order of events, by which for any such decision there exists at least one agent that knows both events, or the decision is not important. Moreover, the third and fourth conditions, guarantee that the cooperative perspective of agents from the tasks (parallel composition of local task automata) neither allows a string that is prohibited by the global task automaton, nor disables a string that is allowed in the global task automaton.

It was furthermore shown that once the task automaton is decomposed into local task automata and local controllers are designed for local plants to satisfy the local specifications, then the global specification is guaranteed, by design.

The next question was the reliability of task decomposability to understand whether a previously decomposable and achievable global task automaton, can still remain decomposable and achievable by the team, after experiencing some event failures. For this purpose, in [21], a class of failures was investigated as follows to define a notion of passivity. Consider an automaton $A = (Q, q_0, E, \delta)$. An event $e \in E$ is said to be failed in A (or E), if $F(A) = P_{\Sigma}(A) = P_{E \setminus e}(A) = (Q, q_0, \Sigma = E \setminus e, \delta^F)$, where, Σ , δ^F and $F(A)$ denote the post-failure event set, post-failure transition relation and post-failure automaton, respectively. A set $\bar{E} \subseteq E$ of events is then said to be failed in A , when for $\forall e \in \bar{E}$, e is failed in A , i.e., $F(A) = P_{\Sigma}(A_i) = P_{E \setminus \bar{E}}(A) = (Q, q_0, \Sigma = E \setminus \bar{E}, \delta^F)$. Considering a parallel distributed plant $A := \parallel_{i=1}^n A_i = (Z, z_0, E = \bigcup_{i=1}^n E_i, \delta_{||})$ with local agents $A_i = (Q_i, q_0^i, E_i, \delta_i)$, $i = 1, \dots, n$. Failure of e in E_i is said to be *passive* in E_i (or A_i) with respect to $\parallel_{i=1}^n A_i$, if $E = \bigcup_{i=1}^n \Sigma_i$. An event whose failure in A_i is a passive failure is called a passive event in A_i .

The passivity was found to reflect the redundancy of communication links and shown to be a necessary condition for preserving the automaton decomposability. It was furthermore shown that when all failed events are passive in the corresponding local event sets, the problem of decomposability under event failure can be transformed into the standard decomposability problem to find the conditions under which $A_S \cong \parallel_{i=1}^n P_{E_i \setminus \bar{E}_i}(A_S)$, as follows.

Lemma 2: (Theorem 1 in [21]) Consider a deterministic task automaton $A_S = (Q, q_0, E = \bigcup_{i=1}^n E_i, \delta)$. Assume that A_S is decomposable, i.e., $A_S \cong \parallel_{i=1}^n P_i(A_S)$, and furthermore, assume that $\bar{E}_i = \{a_{i,r}\}$ fail in E_i , $r \in \{1, \dots, n_i\}$, and \bar{E}_i are passive for $i \in \{1, \dots, n\}$. Then, A_S remains decomposable, in spite of event failures, i.e., $A_S \cong \parallel_{i=1}^n F(P_i(A_S))$ if and only if

- EF1: $\forall e_1, e_2 \in E, q \in Q: [\delta(q, e_1)! \wedge \delta(q, e_2)!]$
 $\Rightarrow [\exists E_i \in \{E_1, \dots, E_n\}, \{e_1, e_2\} \subseteq E_i \setminus \bar{E}_i] \vee [\delta(q, e_1 e_2)! \wedge \delta(q, e_2 e_1)!];$
- EF2: $\forall e_1, e_2 \in E, q \in Q, s \in E^*: [\delta(q, e_1 e_2 s)! \vee \delta(q, e_2 e_1 s)!]$
 $\Rightarrow [\exists E_i \in \{E_1, \dots, E_n\}, \{e_1, e_2\} \subseteq E_i \setminus \bar{E}_i] \vee [\delta(q, e_1 e_2 s)! \wedge \delta(q, e_2 e_1 s)!];$
- EF3: $\delta(q_0, \parallel_{i=1}^n p_i(s_i))!$, $\forall \{s_1, \dots, s_n\} \in \hat{L}(A_S)$, $\exists s_i, s_j \in \{s_1, \dots, s_n\}, s_i \neq s_j$, where,
 $\hat{L}(A_S) \subseteq L(A_S)$ is the largest subset of $L(A_S)$ such that $\forall s \in \hat{L}(A_S), \exists s' \in \hat{L}(A_S), \exists \Sigma_i,$

$\Sigma_j \in \{\Sigma_1, \dots, \Sigma_n\}, i \neq j, p_{\Sigma_i \cap \Sigma_j}(s)$ and $p_{\Sigma_i \cap \Sigma_j}(s')$ start with the same event, and

- *EF4*: $\forall i \in \{1, \dots, n\}, x, x_1, x_2 \in Q_i, x_1 \neq x_2, e \in E_i \setminus \bar{E}_i, t_1, t_2 \in \bar{E}_i^*, \delta_i(x, t_1 e) = x_1, \delta_i(x, t_2 e) = x_2: \delta_i(x_1, t'_1)! \Leftrightarrow \delta_i(x_2, t'_2)!, \text{ for some } t'_1, t'_2 \text{ such that } p_{E_i \setminus \bar{E}_i}(t'_1) = p_{E_i \setminus \bar{E}_i}(t'_2).$

EF1-EF4 are respectively the decomposability conditions *DC1-DC4*, after event failures with respect to parallel composition and natural projections into refined local event sets $\Sigma_i = E_i \setminus \bar{E}_i, i \in \{1, \dots, n\}$, provided passivity of $\bar{E}_i, i \in \{1, \dots, n\}$.

In this paper we are interested in the case that a task automaton is not decomposable and would like to ask whether it is possible to make it decomposable, and if so, whether the automaton can be made decomposable with minimum number of communication links. This problem is formally stated as

Problem 1: Consider a deterministic task automaton A_S with event set $E = \bigcup_{i=1}^n E_i$ for n agents with local event sets $E_i, i = 1, \dots, n$. If A_S is not decomposable, can we modify the sets of private and shared events between local event sets such that A_S becomes decomposable with respect to parallel composition and natural projections P_i , with the minimum number of communication links?

One trivial way to make an automaton A decomposable, is to share all events among all agents, i.e., $E_i = E, \forall i = 1, \dots, n$. This method, however, is equivalent to centralized control. In general, in distributed large scale systems, one of the objectives is to sustain the systems functionalities over as few number of communication links as possible, as will be addressed in the next section.

III. TASK AUTOMATON DECOMPOSABILIZATION

A. Motivating Examples

This section is devoted to Problem 1 and proposes an approach to redefine the set of private and shared events among agents in order to make an undecomposable task automaton decomposable. For more elaboration, let us to start with a motivating examples.

Example 1: Consider two sequential belt conveyors feeding a bin, as depicted in Figure 1. To avoid the overaccumulation of materials on Belt B, when the bin needs to be charged, at first Belt B and then (after a few seconds), Belt A should be started. After filling the bin, to stop the charge, first Belt A and then after a few seconds Belt B is stopped to get completely emptied. The global task automaton, showing the order of events in this plant, is shown in Figure 2.

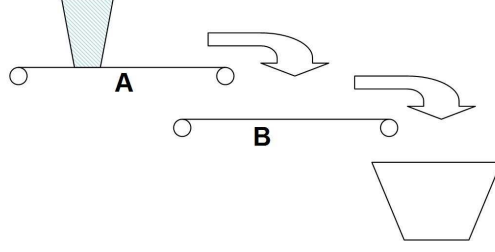


Fig. 1. The process of two belt conveyors charging a bin.

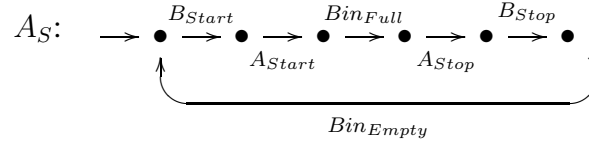


Fig. 2. Global task automaton for belt conveyors and bin.

The local event sets for Belt A and Belt B are $E_A = \{A_{Start}, Bin_{Full}, A_{Stop}\}$ and $E_B = \{B_{Start}, B_{Stop}, Bin_{Empty}\}$, respectively, with $A_{Start} :=$ Belt A start; $Bin_{Full} :=$ Bin full; $A_{Stop} :=$ Belt A stop and wait for 10 Seconds; $B_{Start} :=$ Belt B start and wait for 10 Seconds; $B_{Stop} :=$ Belt B stop, and $Bin_{Empty} :=$ Bin empty.

The task automaton is not decomposable with respect to parallel composition and natural projection P_i , $i \in \{A, B\}$, due to violation of DC by successive private event pairs $\{B_{Start}, A_{Start}\}$ and $\{A_{Stop}, B_{Stop}\}$. To make A_S decomposable, $(B_{Start} \vee A_{Start}) \wedge (A_{Stop} \vee B_{Stop})$ should become common between E_A and E_B . Therefore, four options are possible: $(B_{Start} \wedge B_{Stop})$, $(B_{Start} \wedge A_{Stop})$, $(A_{Start} \wedge B_{Stop})$, or $(A_{Start} \wedge A_{Stop})$ become common. In each of these options two private events should become common, and hence, all four options are equivalent in the sense of optimality. Consider for example A_{Start} and A_{Stop} to become common. In this case the new local event sets are formed as $E_A = \{A_{Start}, Bin_{Full}, A_{Stop}\}$ and $E_B = \{B_{Start}, B_{Stop}, Bin_{Empty}, A_{Start}, A_{Stop}\}$. The automaton A_S will then become decomposable (i.e., $P_A(A_S) || P_B(A_S) \cong A_S$) with the new local event sets with the corresponding local task automata as are shown in Figure 3.

In this example, different sets of private events can be chosen to make A_S decomposable. All of these sets have the same cardinality, and hence, no optimality is arisen in this example. Next example shows a case with different choices of private event sets to be shared, suggesting

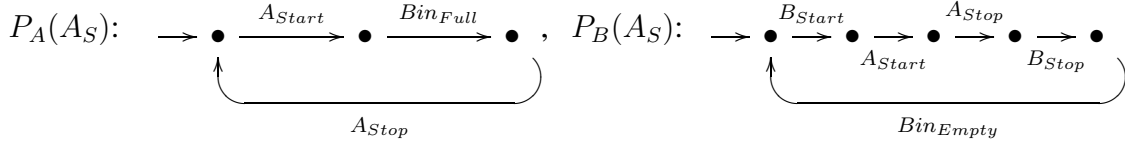


Fig. 3. Local task automata for belt conveyors, with $E_A = \{A_{Start}, Bin_{Full}, A_{Stop}\}$ and $E_B = \{B_{Start}, B_{Stop}, Bin_{Empty}, A_{Start}, A_{Stop}\}$.

optimal decomposition by choosing the set with the minimum cardinality.

Example 2: Consider two local event sets $E_1 = \{e_1, e_3\}$ and $E_2 = \{e_2\}$, with the global task automaton $\longrightarrow \bullet \xrightarrow{e_2} \bullet \xrightarrow{e_3} \bullet$. This automaton is undecomposable due to violation of

DC by $e_2 \in E_2 \setminus E_1$ and $\{e_1, e_3\} \in E_1 \setminus E_2$. To make it decomposable, one event among the set $\{e_1, e_2\}$ and another event among the set $\{e_2, e_3\}$ (either $\{e_2\}$ or $\{e_1, e_3\}$) should become common. Therefore, in order for optimal decomposabilization, $\{e_2\}$ is chosen to become common due to its minimum cardinality. It is obvious that in this case only one event should become common while if $\{e_1, e_3\}$ was chosen, then two events were required to be shared.

Motivated by these examples, the core idea in our decomposabilization approach is to first check the decomposability of a given task automaton A_S , by Lemma 1, and if it is not decomposable, i.e., either of *DC1-DC4* is violated then the proposed method is intended to make A_S decomposable, by eradicating the reasons of dissatisfying of decomposability conditions. We will show that violation of decomposability conditions, can be rooted from two different sources: it can be because of over-communication among agents, that may lead to violation of *DC3* or/and *DC4*, or due to lack of communication, that may lead to violation of *DC1*, *DC2*, *DC3* or/and *DC4*. Accordingly, decomposability can be enforced using two methods of link deletion and link addition, subjected to the type of undecomposability. Considering link deletion as an intentional event failure, according to Lemma 2 a link can be deleted only if it is passive and its deletion respects *EF1-E4*. On the other hand, the second method of enforcing of decomposability, i.e., establishing new communication links, may result in new violations of *DC3* or *DC4*, that should be treated, subsequently.

In order to proceed the approach, we firstly introduce four basic definitions to *detect* the components that contribute in violation of each decomposability condition and then propose basic lemmas through which the communication links, and hence the local event sets are modified to

resolve the violations of decomposability conditions.

B. Enforcing DC1 and DC2

This part deals with enforcing of DC1 and DC2. For this purpose, the set of events that violate DC1 or DC2 is defined as follows.

Definition 1: (DC1&2-Violating set) Consider the global task automaton A_S with local event sets E_i for n agents such that $E = \bigcup_{i=1}^n E_i$. Then, the DC1&2-Violating set operator $V : A_S \rightarrow E \times E$, indicates the set of event pairs that violate DC1 or DC2 (violating pairs), and is defined as $V(A_S) := \{\{e_1, e_2\} | e_1, e_2 \in E, \forall E_i \in \{E_1, \dots, E_n\}, \{e_1, e_2\} \not\subseteq E_i, \exists q \in Q \text{ such that } \delta(q, e_1)! \wedge \delta(q, e_2)! \wedge \neg[\delta(q, e_1 e_2)! \wedge \delta(q, e_2 e_1)!] \text{ or } \neg[\delta(q, e_1 e_2 s)! \Leftrightarrow \delta(q, e_2 e_1 s)!]\}, \text{ for some } s \in E^*\}$. Moreover, $W : A_S \rightarrow E$ is defined as $W(A_S) := \{e \in E | \exists e' \in E \text{ such that } \{e, e'\} \in V(A_S)\}$, and shows the set of events that contribute in $V(A_S)$ (violating events). For a particular event e and a specific local event set $E_i \in \{E_1, \dots, E_n\}$, $W_e(A_S, E_i)$ is defined as $W_e(A_S, E_i) = \{e' \in E_i | \{e, e'\} \in V(A_S)\}$. This set captures the collection of events from E_i that pair up with e to contribute in violation of DC1 or DC2. The cardinality of this set will serve as an index for optimal addition of communication links to make $V(A_S)$ empty.

This definition suggests a way to remove a pair of events $\{e_1, e_2\}$ from $V(A_S)$, by sharing e_1 with one of the agents in $loc(e_2)$ or by sharing e_2 with one of the agents in $loc(e_1)$. Once there exist an agent that knows both event, $loc(e_1) \cap loc(e_2)$ becomes nonempty and e_1 and e_2 no longer contribute in violation of DC1 or DC2 since $[\exists E_i \in \{E_1, \dots, E_n\}, \{e_1, e_2\} \subseteq E_i]$ becomes true for e_1 and e_2 in Lemma 1. Therefore,

Lemma 3: The set $V(A_S)$ becomes empty, if for any $\{e, e'\} \in V(A_S)$, e is included in E_i for some $i \in loc(e')$, or e' is included in E_j for some $j \in loc(e)$. In this case, $\{e, E_i\}$ or $\{e', E_j\}$ is called a DC1&2-enforcing pair for DC1&2-violating pair $\{e, e'\}$.

Example 3: In Example 2, $V(A_S) = \{\{e_1, e_2\}, \{e_2, e_3\}\}$, $W(A_S) = \{e_1, e_2, e_3\}$. Including e_2 in E_1 vanishes $V(A_S)$ and makes A_S decomposable.

However, applying Lemma 3 may offer different options for event sharing, since pairs in $V(A_S)$ may share some events. In this case, the minimum number of event conversions would be obtained by forming a set of events that are most frequently shared between the violating pairs. This gives the minimum cardinality for the set of private events to be shared, leading to minimum number of added communication links. Such choice of events offers a set of events

that span all violating pairs. These pairs are captured by $W_e(A_S, E_i)$ for any event e . In order to minimize the number of added communication links for vanishing $V(A_S)$, one needs to maximize the number of deletions of pairs from $V(A_S)$ per any link addition. For this purpose, for any event e , $W_e(A_S, E_i)$ is formed to understand the frequency of appearance of e in $V(A_S)$ for any E_i , and then, the event set E_i with maximum $|W_e(A_S, E_i)|$ is chosen to include e (Here, $|\cdot|$ denotes the set's cardinality). In this case, inclusion of e in E_i will delete as many pairs as possible from $V(A_S)$.

Interestingly, these operators can be represented using graph theory as follows. A graph $G = (W, \Sigma)$ consists of a node set W and an edge set Σ , where an edge is an unordered pair of distinct vertices. Two nodes are said to be adjacent if they are connected through an edge, and an edge is said to be incident to a node if they are connected. The valency of a node is then defined as the number of its incident edges [33]. Now, since we are interested in removing the violating pairs by making one of their events to be shared, it is possible to consider the violating events as nodes of a graph such that two nodes are adjacent in this graph when they form a violating pair. This graph is formally defined as follows.

Definition 2: (DC1&2-Violating Graph) Consider a deterministic automaton A_S . The DC1&2-Violating graph, corresponding to $V(A_S)$, is a graph $G(A_S) = (W(A_S), \Sigma)$. Two nodes e_1 and e_2 are adjacent in this graph when $\{e_1, e_2\} \in V(A_S)$.

In this formulation, the valency of each node e with respect to a local event set $E_i \in \{E_1, \dots, E_n\}$ is determined by $val(e, E_i) = |W_e(A_S, E_i)|$. When e is included into E_i , it means that all violating pairs containing e and events from E_i are removed from $V(A_S)$, and equivalently, all corresponding incident edges are removed from $G(A_S)$. For this purpose, following algorithm finds the set with the minimum number of private events to be shared, in order to satisfy DC1 and DC2. The algorithm is accomplished on graph $G(A_S)$, by finding e and E_i with maximum $|W_e(A_S, E_i)|$ and including e in E_i , deleting all edges from e to E_i , updating $W(A_S)$, and continuing until there is not more edges in $G(A_S)$ to be deleted.

Algorithm 1:

- 1) For a deterministic automaton A_S , with local event sets $E_i, i = 1, \dots, n$, violating DC1 or DC2, form the DC1&2-Violating graph ; set $E_i^0 = E_i, i = 1, \dots, n; V^0(A_S) = V(A_S); W^0(A_S) = W(A_S); G^0(A_S) = (W(A_S), \Sigma); k=1;$
- 2) Among all events in the nodes in $W^{k-1}(A_S)$, find e with the maximum $|W_e^{k-1}(A_S, E_i^{k-1})|,$

- for all $E_i^{k-1} \in \{E_1^{k-1}, \dots, E_n^{k-1}\}$;
- 3) $E_i^k = E_i^{k-1} \cup \{e\}$; and delete all edges from e to E_i^k ;
 - 4) update $W_e^k(A_S, E_i)$ for all nodes of $G(A_S)$;
 - 5) set $k = k + 1$ and go to step (2);
 - 6) continue, until there exist no edges.

This algorithm successfully terminates due to finite set of edges and nodes in the graph $G(A_S)$ and enforces A_S to satisfy $DC1$ and $DC2$ as

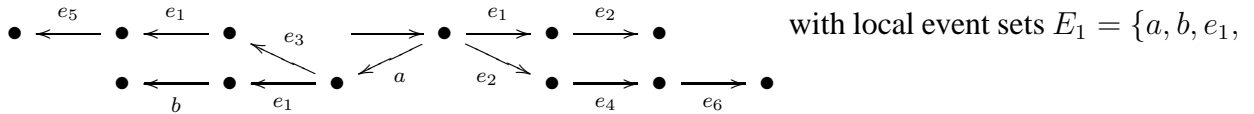
Lemma 4: Algorithm 1 leads A_S to satisfy $DC1$ and $DC2$ with minimum addition of communication links. Moreover if A_S satisfies $DC3$ and $DC4$ and $E_i^k = E_i^{k-1} \cup \{e\}$ in Step 3 does not violate $DC3$ and $DC4$ in all iterations, then Algorithm 1 makes A_S decomposable with minimum addition of communication links.

Proof: See the Appendix for proof. ■

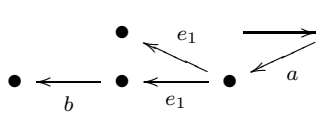
Remark 1: (Special case: Two agents) For the case of two agents, since they are only two local event sets, for all $\{e, e'\} \in V(A_S)$, e and e' are from different local event sets, and hence, for $n = 2$, $|W_e(A_S, E_i)|$ is equivalent to $val(e)$, and addition of e into E_i in each step implies the deletion of all incident edges of e .

Remark 2: Although Algorithm 1 leads A_S to satisfy $DC1$ and $DC2$, it may cause new violations of $DC3$ or/and $DC4$, due to establishing new communication links.


Example 4: Consider a task automaton A_S :



Both $DC1$ and $DC2$ are violated by event pair $\{e_1, e_2\}$ when they require decision on a choice and a decision on their order from the initial state, while none of the agents knows both of them. To vanish $V(A_S) = \{\{e_1, e_2\}\}$, two enforcing pairs are suggested: $\{e_1, E_2\}$ (e_1 to be included in E_2) or $\{e_2, E_1\}$ (e_2 to be included in E_1). However, inclusion of e_1 in E_2 , cause a new violation of $DC4$ since with new $E_2 = \{a, b, e_1, e_2, e_4, e_6\}$, $P_2(A_S)$ is obtained as $P_2(A_S)$:



for which e_3 also is required to be included to E_2 in order to make A_S decomposable. On the other hand, if instead of including e_1 in E_2 , one included e_2 in E_1 , then besides violation of


 produces string $e_1 e_2 e_4 e_6$ that does not appear

in A_S . To make A_S decomposable, we also need to include e_1 and e_3 in E_2 .

Lemma 3 proposes adding communication links to make $DC1$ and $DC2$ satisfied. Next step is to deal with violations of $DC3$. In contrast to the cases for $DC1$ and $DC2$, violation of $DC3$ can be overcome either by disconnecting one of its communication links to prevent the illegal synchronization of strings, or by introducing new shared events to fix strings and avoid illegal interleavings.

Definition 3: (DC3 – violating tuples) Consider a deterministic automaton A_S , satisfying *DC1* and *DC2* and let $\tilde{L}(A_S) \subseteq L(A_S)$ be the largest subset of $L(A_S)$ such that $\forall s \in \tilde{L}(A_S) \exists s' \in \tilde{L}(A_S), \exists E_i, E_j \in \{E_1, \dots, E_n\}, i \neq j, p_{E_i \cap E_j}(s)$ and $p_{E_i \cap E_j}(s')$ start with the same event $a \in E_i \cap E_j$. For any such E_i, E_j and a , if $\exists \{s_1, \dots, s_n\} \in L(A_S), \exists s_i, s_j \in \{s_1, \dots, s_n\}, s_i \neq s_j, s_i, s_j \in \tilde{L}(A_S), \neg \delta(q_0, \bigcup_{i=1}^n p_i(s_i))!$, then a is called a *DC3 – violating event* with respect to s_1, s_2, E_i and E_j , and (s_1, s_2, a, E_i, E_j) is called a *DC3-violating tuple*. The set of all *DC3 – violating tuples* is denoted by *DC3 – V* and defined as $DC3 - V = \{(s_1, s_2, a, E_i, E_j) | e \text{ is a DC3-violating event with respect to } s_1, s_2, E_i \text{ and } E_j\}$.

Any violation in *DC3* can be interpreted in two ways: firstly, it can be seen as over-communication of shared event a that lead to synchronization of s_1 and s_2 in (s_1, s_2, a, E_i, E_j) and emerging illegal interleaving strings from composition of $P_i(A_S)$ and $P_j(A_S)$. In this case, if event a is excluded from E_i or E_j , then a will no longer contribute in synchronization to generate illegal interleavings, and hence, (s_1, s_2, a, E_i, E_j) will no longer remain a *DC3*-violating tuple. However, exclusion of a from E_i or E_j is allowed, only if it is passive (exclusion is considered as an intentional event failure) and does not violate *EF1-EF4*. The second interpretation reflects a violation of *DC3* as a lack of communication, such that if for any *DC3* violating tuple

(s_1, s_2, a, E_i, E_j) , one event that appears before a in s_1 or s_2 , is shared between E_i and E_j , then $P_i(A_S)$ and $P_j(A_S)$ will have enough information to distinguish s_1 and s_2 to prevent illegal interleaving of strings. Two methods for resolving the violation of $DC3$ can be therefore stated as the following lemma.

Lemma 5: Consider an automaton A_S , satisfying $DC1$ and $DC2$. Then any $DC3$ -violating tuple (s_1, s_2, a, E_i, E_j) is overcome, when:

- 1) a is excluded from E_i or E_j (eligible if it respects passivity and $EF1$ - $EF4$), or
- 2) if $\exists b \in (E_i \cup E_j) \setminus (E_i \cap E_j)$ that appears before a in only one of s_1 and s_2 , then b is included in $E_i \cap E_j$, otherwise, pick $e_1 \in p_{E_i \cup E_j}(s_1)$, $e_2 \in p_{E_i \cup E_j}(s_2)$, such that $e_1 \neq e_2$, e_1, e_2 appear before a in s_1 and s_2 , are included in $E_i \cap E_j$.

To handle a violation of $DC3$, when, $b \in E_i \setminus E_j$ is to be included in E_j , then $\{b, E_j\}$ is called a $DC3$ -enforcing pair; while, when $\{e_1, e_2\} \subseteq E_i \setminus E_j$ has to be included in E_j , then $\{\{e_1, e_2\}, E_j\}$ is denoted as $DC3$ -enforcing tuple. Finally, when $e_1 \in E_i \setminus E_j$ and $e_2 \in E_j \setminus E_i$ have to be included in E_j and E_i , respectively, then $\{\{e_1, E_j\}, \{e_2, E_i\}\}$ is called a $DC3$ -enforcing tuple.

Proof: See the proof in the Appendix. ■

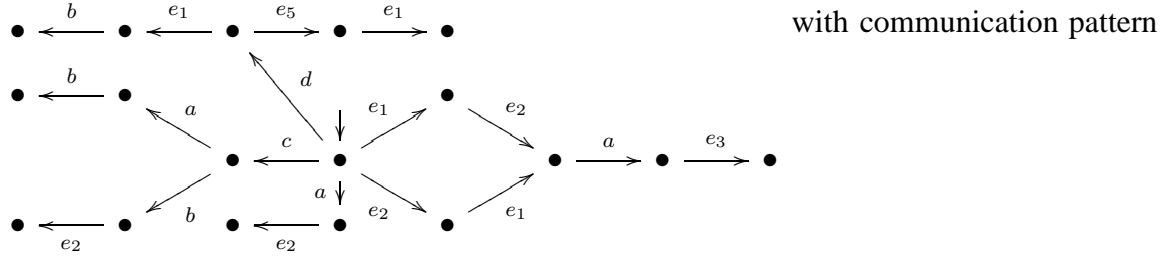
Remark 3: Applying the first method in Lemma 5, namely, exclusion of a from E_i or E_j in a $DC3$ -violating tuple (s_1, s_2, a, E_i, E_j) , is only allowed if a is passive in that local event set, and the exclusion does not violate $EF1$ - $EF4$. The reason is that once a shared event $a \in E_i \cap E_j$ becomes a private one in for example E_i , then decision makings on the order/selection between any $e \in E_i \setminus a$ and a cannot be accomplished by the i -th agent, and if there is no other agent to do so, then A_S becomes undecomposable. Moreover, deletion of a communication link may also result in generation of new interleavings in the composition of local automata, that are not legal in A_S (violation of $EF3$). In addition, deletion of a from E_i may impose a nondeterminism in bisimulation quotient of $P_i(A_S)$, leading to violation of $EF4$. On the other hand, the second method, namely, establishing new communication link by sharing b with E_i or E_j may lead to new violations of $DC3$ or $DC4$ that have to be avoided or resolved, subsequently.

Both methods in Lemmas 5 present ways to resolve the violation of $DC3$. They differ however in the number of added communication links, as the first method deletes links, whereas the second approach adds communication links to enforce $DC3$. Therefore, in order to have as few number of links as possible among the agents, one should start with the link deletion method first, and

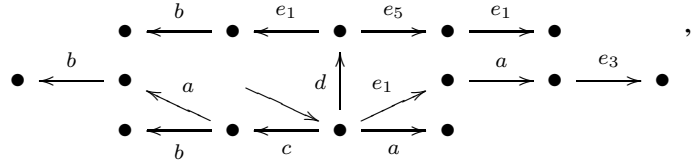
if it is not successful due to violation of passivity or any of $EF1$ - $EF4$, then link addition is used to remove $DC3$ -violating tuples from $DC3 - V$.

Example 5: This example shows an undecomposable automaton that suffers from a conflict on a communication link whose existence violates $DC3$, whereas its deletion dissatisfies $EF1$, $EF2$ and $EF4$.

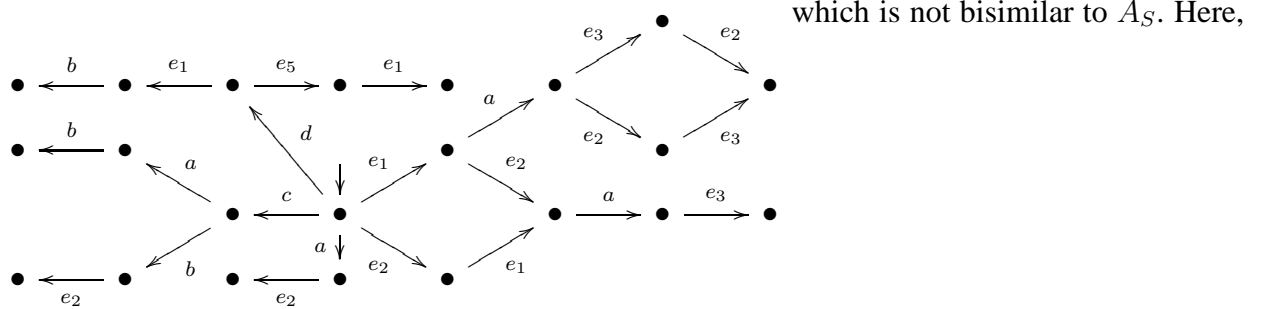
Let $snd_e(i)$ and $rcv_e(i)$ respectively denote the set of labels that A_i sends e to those agents and the set of labels that A_i receives e from their agents, defined as $snd_e(i) = \{j \in \{1, \dots, n\} | A_i \text{ sends } e \text{ to } A_j\}$ and $rcv_e(i) = \{j \in \{1, \dots, n\} | i \in snd_e(j)\}$. Consider the task automaton A_S :



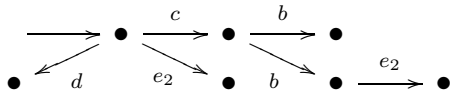
$2 \in snd_{a,b,c,d}(1)$, $1 \notin snd_{a,b,c,d}(1)$ and local event sets $E_1 = \{a, b, c, d, e_1, e_3, e_5\}$, $E_2 = \{a, b, c, d, e_2\}$, leading to $P_1(A_S)$:



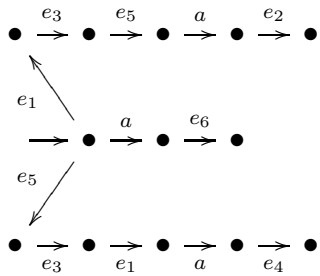
$P_2(A_S)$: and $P_1(A_S) || P_2(A_S)$:

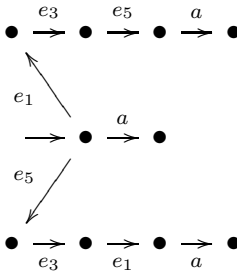
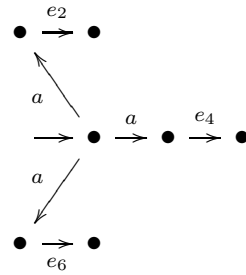


A_S is not decomposable since two strings $e_1ae_2e_3$ and $e_1ae_3e_2$ are newly generated from the interleaving of strings in $P_1(A_S)$ and $P_2(A_S)$, while they do not appear in A_S , and hence, $DC3$ is not fulfilled, due to $DC3$ -violating tuples $(e_1e_2ae_3, ae_2, a, E_1, E_2)$ and $(e_2e_1ae_3, ae_2, a, E_1, E_2)$. Now, as Lemma 5, one way to fix the violation of $DC3$ is by excluding a from E_2 . However, although a is passive in E_2 , its exclusion from E_2 dissatisfies $EF1$ (as $\delta(q_0, e_2)! \wedge \delta(q_0, a)! \wedge \neg[\delta(q_0, e_2a)! \wedge \delta(q_0, ae_2)!]$) and $EF2$ (since $\delta(q_0, e_1e_2a)! \wedge \neg\delta(q_0, e_1ae_2)!$). In this case, $DC4$

also will be violated as $P_2(A_S)$ becomes $P_2(A_S) \cong$  that bisimulates no deterministic automaton.

Lemma 5 also suggests another method to enforce *DC3*, by including either e_1 in E_2 or e_2 in E_1 . Inclusion of e_1 in E_2 , however, leads to another violation of *DC4*, as it produces a non-determinism after event d . This in turn will need to include e_5 in E_2 to make A_S decomposable. Alternatively, instead of inclusion of e_1 in E_2 , one can include e_2 in E_1 , that enforces *DC3* and makes A_S decomposable. The second method of Lemma 5 is more elaborated in the next example.

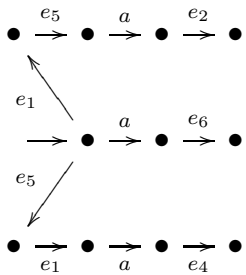
Example 6: This example shows handling of *DC3*-violating tuples using the second method in Lemma 5, i.e., by event sharing. Later on, this example will be also used to illustrate the enforcement of *DC4*. Now, consider a task automaton A_S :  with local

event sets $E_1 = \{a, e_1, e_3, e_5\}$ and $E_2 = \{a, e_2, e_4, e_6\}$, and let three branches in A_S from top to bottom to be denoted as $s_1 := e_1e_3e_5ae_2$, $s_3 := ae_6$ and $s_2 := e_5e_3e_1ae_4$. This automaton does not satisfy *DC4* (as $P_2(A_S)$ has no deterministic bisimilar automaton), as well as *DC3*, as the parallel composition of $P_1(A_S)$:  and $P_2(A_S)$:  have

illegal interleaving strings $\{e_1e_3e_5ae_6, e_5e_3e_1ae_2\}$, $e_1e_3e_5ae_4$ and $e_5e_3e_1ae_4$, corresponding to *DC3*-violating tuples (s_1, s_2, a, E_1, E_2) , (s_1, s_3, a, E_1, E_2) and (s_2, s_3, a, E_1, E_2) , respectively.

For pairs of strings $\{s_1, s_3\}$ and $\{s_2, s_3\}$, there exists an event $e_5 \in (E_1 \cup E_2) \setminus (E_1 \cap E_2)$ that appears before a , only in s_1 and s_2 , but not in s_3 . Therefore, inclusion of e_5 in E_2 , removes the illegal interleavings between s_1 and s_2 with s_3 , but not across s_1 and s_2 , as with new $E_2 =$

$\{a, e_2, e_4, e_5, e_6\}$ and $P_2(A_S)$:  , (s_1, s_3, a, E_1, E_2) and (s_2, s_3, a, E_1, E_2) are

no longer *DC3*-violating tuples, while (s_1, s_2, a, E_1, E_2) still remains a *DC3*-violating one with illegal interleavings $e_1e_3e_5ae_4$ and $e_5e_3e_1ae_2$. The reason is that e_5 appears before a in both s_1 and s_2 , and there is no event that appear before a only in one of the strings s_1 and s_2 . For this case, according to Lemma 5, two different events that appear before “ a ”, one from $p_{E_1 \cup E_2}(s_1) = s_1$ and the other from $p_{E_1 \cup E_2}(s_2) = s_2$, i.e., e_1 and e_5 have to be attached to E_2 , resulting in $E_2 = \{a, e_1, e_2, e_4, e_5, e_6\}$,  and $P_1(A_S) || P_2(A_S) \cong A_S$.

D. Enforcing *DC4*

Similar to *DC1-DC3*, a violation of *DC4* can be regarded as a lack of communication link that causes nondeterminism in a local task automaton. Such interpretation calls for establishing a new communication link to prevent the emergence of local nondeterminism. Moreover, when this local nondeterminism occurs on a shared event, the corresponding violation of *DC4* can be overcome by excluding the shared event from the respective local event set. It should be noted however that the event exclusion should respect the passivity and *EF1-EF4* conditions. When *DC4* is enforced by link additions, similar to what we discussed for *DC3*, addition of new communication link may cause new violations of *DC3* or/and *DC4*. To enforce *DC4*, firstly a *DC4*-violating tuple is defined as follows.

Definition 4: (DC4 – violating tuple) Consider a deterministic automaton A_S with local event sets $E_i = 1, \dots, n, \forall i \in \{1, \dots, n\}$, $q, q_1, q_2 \in Q$, $t_1, t_2 \in (E \setminus E_i)^*$, $e \in E_i$, $\delta(q, t_1e) = q_1 \neq \delta(q, t_2e) = q_2$, $\exists t \in E^*$, $\delta(q_1, t)!$, but $\nexists t' \in E^*$ such that $\delta(q_2, t')!$, $p_i(t) = p_i(t')$. Then, (q, t_1, t_2, e, E_i) is called a *DC4*-violating tuple.

This definition suggests the way to overcome the violation of $DC4$, as stated in the following lemma.

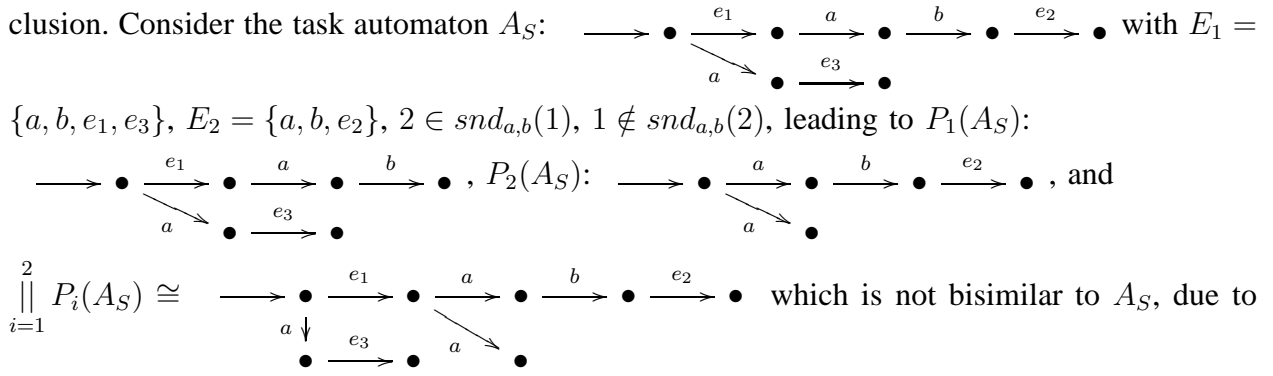
Lemma 6: Any $DC4$ -violating tuple (q, t_1, t_2, e, E_i) is overcome, when:

- 1) e is excluded from E_i , (eligible, if it is passive in E_i and its exclusion respects $EF1-EF4$),
or
- 2) if $\exists e' \in (t_1 \cup t_2) \setminus (t_1 \cap t_2)$, e' is included in E_i ; otherwise, $e_1 \in t_1$ and $e_2 \in t_2$, such that $e_1 \neq e_2$, are included in E_i . In these cases, $\{e', E_i\}$ and $\{\{e_1, e_2\}, E_i\}$ are called $DC4$ -enforcing tuples.

Proof: See the proof in the Appendix. ■

Following examples illustrate the methods in Lemma 6 to enforce $DC4$.

Example 7: This example shows an automaton that is undecomposable due to a violation in $DC4$, while $DC4$ can be enforced using both methods: event exclusion as well as event inclusion. Consider the task automaton A_S :



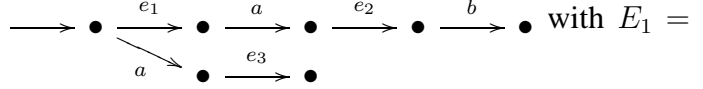
violation of $DC4$ as there does not exist a deterministic automaton $P'_2(A_S)$ such that $P'_2(A_S) \cong P_2(A_S)$. Here, $(q_0, t_1 = e_1, t_2 = \varepsilon, a, E_2)$ is a $DC4$ -violating tuple. Since a is passive in E_2 and its exclusion from E_2 keeps $EF1-EF4$ valid, according to Lemma 6, one way to enforce $DC4$ is exclusion of a from E_2 , resulting in $E_2 = \{b, e_2\}$, $P_2(A_S)$: $\longrightarrow \bullet \xrightarrow{b} \bullet \xrightarrow{e_2} \bullet$ and $P_1(A_S) \parallel P_2(A_S) \cong A_S$.

Another suggestion of Lemma 6 to overcome the $DC4$ -violating tuple $(q_0, t_1 = e_1, t_2 = \varepsilon, a, E_2)$ is addition of a communication link to prevent the nondeterminism in $P_2(A_S)$. Since there exists e_1 that appears before a in t_1 only, inclusion of e_1 in E_2 also enforces $DC4$ as with new $E_2 = \{a, b, e_1, e_2\}$, $P_2(A_S)$: $\longrightarrow \bullet \xrightarrow{e_1} \bullet \xrightarrow{a} \bullet \xrightarrow{b} \bullet$ and $\bigparallel_{i=1}^2 P_i(A_S) \cong A_S$. For

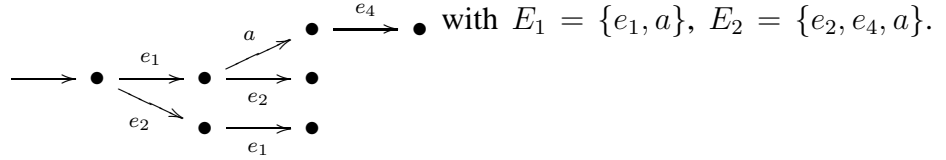
the cases that there does not exist an event b that appears before a in only one of the strings t_1 or t_2 , according to Lemma 6, one needs to attach one event from each of two strings t_1 and t_2 in

E_i . For instance consider the $DC4$ -violating tuple $(t_1 = e_1e_3e_5, t_2 = e_5e_3e_1, a, E_2)$ in Example 6, with no event that appears before a in $(t_1 \cup t_2) \setminus (t_1 \cap t_2)$. In that case $\{e_1 \in t_1, e_5 \in t_2\}$ can be included in E_2 to make A_S decomposable, as it was shown in Example 6.

Example 8: Example 7 showed a violation of $DC4$ that could be overcome using both method in Lemma 6, namely, by link deletion and link addition. In Example 7, event a was a passive shared event whose exclusion from E_2 respected $EF1$ - $EF4$, otherwise it was not allowed to be excluded. If the task automaton was



with $E_1 = \{a, b, e_1, e_3\}$, $E_2 = \{a, b, e_2\}$, then $DC4$ could not be enforced by exclusion of a from E_2 , as $EF2$ was violated since after this exclusion, no agent can handle the decision making on the order of a and e_2 . Another constraint for link deletion is the passivity of the event. For example, consider A'_S :

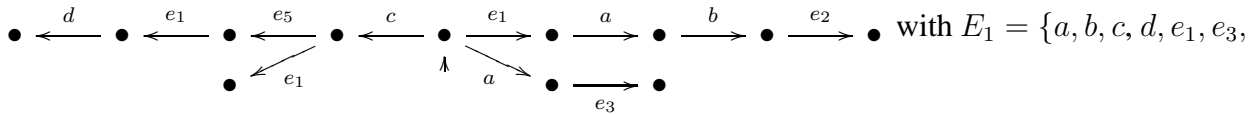


A'_S is not decomposable due to violation of $DC4$ in $P_1(A_S)$: . The

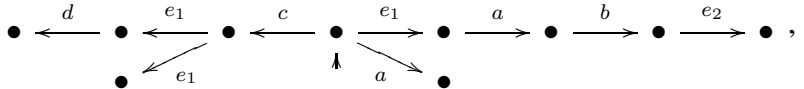
nondeterminism in $P_1(A_S)$, and accordingly the $DC4$ -violating tuple $(q_0, \varepsilon, e_2, e_1, E_1)$, cannot be removed by event exclusion since it occurs on e_1 that is not a shared event. To enforce $DC4$ according to Lemma 6, e_2 is required to be included into E_1 that makes A'_S decomposable.

Another important issue for addition of communication link to enforce $DC4$ is that establishing new communication link may lead to new violations of $DC3$ or $DC4$, as it is shown in the following example.

Example 9: Assume the task automaton in Example 7 had a part as shown in the left hand side of the initial state in A_S :

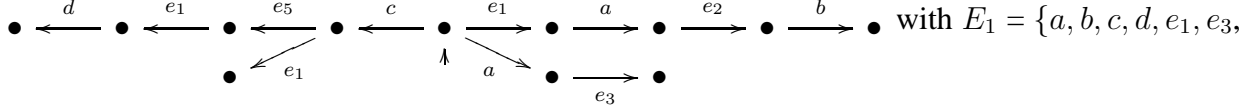


with $E_1 = \{a, b, c, d, e_1, e_3, e_5\}$, $E_2 = \{a, b, c, d, e_2\}$. Identical to Example 7, $(q_0, t_1 = e_1, t_2 = \varepsilon, a, E_2)$ is a $DC4$ -violating tuple and can be overcome by excluding a from E_2 , removing the nondeterminism on a in $P_2(A_S)$. However, unlike Example 7, including e_1 into E_2 (i.e., $E_2 = \{a, b, c, d, e_1, e_2\}$), leads to a new violation of $DC4$ in $P_2(A_S)$:



with a $DC4$ -violating tuple $(\delta(q_0, c), e_5, \varepsilon, e_1, E_2)$, that in turn requires attachment of e_5 to E_2 , in order to enforce $DC4$.

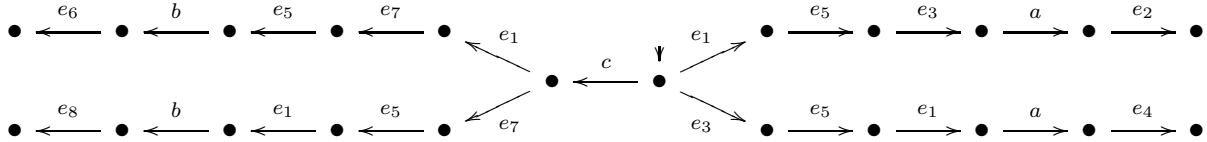
If in this example, the order of e_2 and b was reverse, i.e., the task automaton was A'_S :



$e_5\}$, $E_2 = \{a, b, c, d, e_2\}$. Then as it was shown in Example 8, the $DC4$ -violating tuple $(q_0, e_1, \varepsilon, a, E_2)$ could not be dealt with exclusion of a from E_2 , due to $EF2$, neither by inclusion of e_1 into E_2 (since as mentioned above, it generates a new violation of $DC4$ that consequently requires another inclusion of e_5 into E_2 to satisfy $DC4$).

Remark 4: Both Lemmas 5 and 6 provide sufficient conditions for resolving the violations of $DC3$ and $DC4$, respectively. They do not however provide the necessary solutions, neither the optimal solutions, as illustrated in the following example. We will show that for $DC3$ and $DC4$, in general one requires to search exhaustively to find the optimal sequence of enforcing tuples, to have minimum number of link additions. In this sense, instead of exhaustive search for optimal solution, it is reasonable to introduce sufficient conditions to provide a trackable procedure for a feasible solution to make an automaton decomposable.

Example 10: Consider a task automaton A_S :



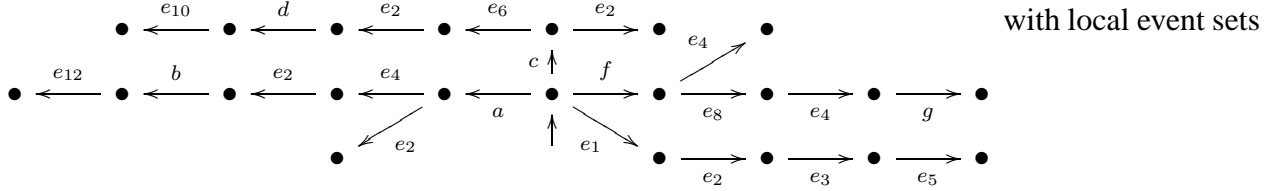
with local event sets $E_1 = \{a, b, c, e_1, e_3, e_5, e_7\}$ and $E_2 = \{a, b, c, e_2, e_4, e_6, e_8\}$. A_S is undecomposable due to $DC3$ -violating tuples $(e_1e_5e_3ae_2, e_3e_5e_1ae_4, a, E_1, E_2)$ and $(e_1e_7e_5be_6, e_7e_5e_1be_8, a, E_1, E_2)$ and $DC4$ -violating tuples $(q_0, e_1e_5e_3, e_3e_5e_1, a, E_2)$ and $(\delta(q_0, c), e_1e_7e_5, e_7e_5e_1, b, E_2)$. According to Lemmas 5 and 6, two enforcing tuples $\{\{e_1, e_3\}, E_2\}$ and $\{\{e_1, e_7\}, E_2\}$ remove all violations of $DC3$ and $DC4$. However, this solution is not unique, nor optimal, as the enforcing tuple $\{\{e_1, e_5\}, E_2\}$ enforced $DC3$ and $DC4$ with minimum number of added communication links.

E. Exhaustive search for optimal decompozabilization

Another difficulty is that enforcing the decomposability conditions using link deletion is limited to passivity and $EF1$ - $EF4$, and after deletions of redundant links (that are passive and their

deletion respect *EF1-EF4*), the only way to make the automaton decomposable is to establish new communication links. Addition of new links, on the other hand, may lead to new violations of *DC3* or *DC4* (as illustrated in Examples 5 and 9), and in turn may introduce new violations. It means that, in general, resolution of decomposability conditions can dynamically result in new violations of decomposability conditions, as it is elaborated in the following example.

Example 11: Consider the task automaton A_S :



$E_1 = \{a, b, c, d, f, g, e_1, e_3, e_5\}$ and $E_2 = \{a, b, c, d, f, g, e_2, e_4, e_6, e_8, e_{10}, e_{12}\}$. This automaton is undecomposable due to *DC2*-violating event pairs $\{(e_1, e_2), (e_2, e_3)\}$ with the corresponding enforcing tuples $\{e_1, E_2\}$, $\{e_3, E_2\}$ and $\{e_2, E_1\}$ and with the following possible sequences:

- 1) $\{e_1, E_2\}; \{e_3, E_2\}$: in this case A_S becomes decomposable, without emerging new violations of decomposability conditions;
- 2) $\{e_1, E_2\}; \{e_2, E_1\}; \{\{e_4, e_6\}, E_1\}; \{e_8, E_1\}$: if after including e_1 in E_2 , e_2 is included in E_1 , then two *DC4*-violating tuples $(\delta(q_0, a), \varepsilon, e_4, e_2, E_1)$ and $(\delta(q_0, c), \varepsilon, e_6, e_2, E_1)$ emerge that in turn require $\{e_4, e_6\}$ to be attached to E_1 . Inclusion of e_4 in E_1 , on the other hand, introduces another *DC4*-violating tuple $(\delta(q_0, f), \varepsilon, e_8, e_4, E_1)$ that calls for attachment of e_8 to E_1 ; similarly
- 3) $\{e_3, E_2\}; \{e_1, E_2\}$;
- 4) $\{e_3, E_2\}; \{e_2, E_1\}; \{\{e_4, e_6\}, E_1\}; \{e_8, E_1\}$, and
- 5) $\{e_2, E_1\}; \{\{e_4, e_6\}, E_1\}; \{e_8, E_1\}$.

In this example, the first and the third sequences, i.e., $\{\{e_1, e_3\}, E_2\}$ gives the optimal choice with minimum number of added communication links, although initially $\{e_2, E_1\}$ sought to offer the optimal solution.

Therefore, in general an optimal solution to Problem 1 will be obtained through an exhaustive search, using Lemmas 4, 5 and 6, as state in the following algorithm.

Algorithm 2:

- 1) For any local event set, exclude any passive event whose exclusion respects *EF1-EF4*;
- 2) identify all *DC1&2*-violating tuples, *DC3*-violating tuples and *DC4*-violating tuples and

their respective enforcing tuples;

- 3) among all enforcing tuples, find the one that corresponds to the most violating tuples;
- 4) if applying of the enforcing tuples with maximum number of violating tuples, does not impose new violations of $DC3$ or $DC4$, then apply it, go to Step 3 and continue until there is no violating tuples; otherwise, do the exhaustive search to find the sequence of link additions with minimum number of added links.
- 5) end.

Lemma 7: Consider a deterministic task automaton A_S with local event sets E_i such that $E = \bigcup_{i=1}^n E_i$. If A_S is not decomposable with respect to parallel composition and natural projections P_i , $i = 1, \dots, n$, Algorithm 2 optimally makes A_S decomposable, with minimum number of communication links.

Proof: See the proof in the Attachment. ■

Remark 5: (Special case: Automata with mutual exclusive branches) When branches of A_S share no events (i.e. $\forall q \in Q, s, s' \in E^*, \delta(q, s)!, \delta(q, s')!, s \not\leq s', s' \not\leq s: s \cap s' = \emptyset$), due to definition of $DC3$ and $DC4$ in Lemma 1 $DC3$ and $DC4$ are trivially satisfied, and moreover, since branches from any state share no event, then Algorithm 2 is reduced to Algorithm 1.

F. Feasible solution for task decomposabilization

As Example 11 showed that, in general, additions of communication links may successively introduce new violations of decomposability conditions, for which new links should be established. Therefore, in general an optimal solution to Problem 1 requires an exhaustive search, using Lemmas 4, 5 and 6. Moreover, checking of $DC3$ and $DC4$ is a nontrivial task, while it has to be accomplished initially as well as upon each link addition. It would be therefore very tractable if we can define a procedure to make $DC3$ and $DC4$ satisfied, without their examination. Following result takes an automaton whose $DC1$ and $DC2$ are made satisfied using Algorithm 1, and proposes a sufficient condition to fulfill $DC3$ and $DC4$.

Lemma 8: Consider a deterministic automaton A_S , satisfying $DC1$ and $DC2$. A_S satisfies $DC3$ and $DC4$ if following steps are accomplished on A_S :

- 1) $\forall s_1, s_2 \in E^*, s_1 \not\leq s_2, s_2 \not\leq s_1, q, q_1, q_2 \in Q, \delta(q, s_1) = q_1 \neq \delta(q, s_2) = q_2, [\nexists e_1, e_2 \in E, e_1 e_2 \leq s_1, e_2 e_1 \leq s_2, \forall t \in E^*, \delta(q, e_1 e_2 t)! \Leftrightarrow \delta(q, e_2 e_1 t)!], \exists e \in s_1 \cap s_2, \text{ then } \forall i \in \text{loc}(e), \forall e' \in \{e_1 \leq t_1, e_2 \leq t_2\}, e' \text{ appears before } e, \text{ include } e' \text{ in } E_i.$

- 2) go to Step 1 and continue until $\forall s_1, s_2 \in E^*, s_1 \not\leq s_2, s_2 \not\leq s_1, q, q_1, q_2 \in Q, \delta(q, s_1) = q_1 \neq \delta(q, s_2) = q_2, \exists e \in s_1 \cap s_2, [\nexists e_1, e_2 \in E, e_1 e_2 \leq s_1, e_2 e_1 \leq s_2, \forall t \in E^*, \delta(q, e_1 e_2 t)! \Leftrightarrow \delta(q, e_2 e_1 t)!]$, then $\forall i \in \text{loc}(e), E_i$ contains the first events of s_1 and s_2 , that appear before e .

Proof: See the proof in the Attachment. ■

Remark 6: The condition in Lemma 8 intuitively means that for any two strings s_1, s_2 from any state q , sharing an event e , all agents who know this event e will be able to distinguish two strings, if they know the first event of each string. The ability of those agents that know this event e to distinguish strings s_1 and s_2 , prevents illegal interleavings (to enforce *DC3*) and local nondeterminism (to satisfy *DC4*). The significance of this condition is that it does not require to check *DC3* and *DC4*, instead provides a tractable (but more conservative) procedure to enforce *DC3* and *DC4*. The expression $s_1 \not\leq s_2, s_2 \not\leq s_1$ in the lemma, is to exclude the pairs of strings that one of them is a substring of the other, as their language product does not exceed from the strings of A_S , provided *DC1* and *DC2*. Moreover, the expression $[\nexists e_1, e_2 \in E, e_1 e_2 \leq s_1, e_2 e_1 \leq s_2, \forall t \in E^*, \delta(q, e_1 e_2 t)! \Leftrightarrow \delta(q, e_2 e_1 t)!]$ in this lemma excludes the pairs of strings $e_1 e_2 t$ and $e_2 e_1 t$ from any $q \in Q$ that have been already checked using *DC1* and *DC2* and do not form illegal interleaving strings, and hance, do not need to include e_1 in the local event sets of e_2 and vice versa (see Example 12).

Combination of Lemmas 4 and 8 leads to the following algorithm as a sufficient condition to make a deterministic task automaton decomposable. Following algorithm uses Lemma 4 to enforce *DC1* and *DC2* followed by Lemma 8 to overcome the violations of *DC3* and *DC4*.

Algorithm 3:

- 1) For a deterministic automaton A_S , with local event sets $E_i, i = 1, \dots, n, \forall E_i \in \{E_1, \dots, E_n\}$, $E_i^0 = E_i \setminus \{e \in E_i | e \text{ is passive in } E_i \text{ and exclusion of } e \text{ from } E_i \text{ does not violate } EF1\text{-}EF4\}$;
- 2) form the *DC1&2-Violating* graph ; set $V^0(A_S) = V(A_S); W^0(A_S) = W(A_S); G^0(A_S) = (W(A_S), \Sigma); k=1$;
- 3) Among all events in the nodes in $W^{k-1}(A_S)$, find e with the maximum $|W_e^{k-1}(A_S, E_i^{k-1})|$, for all $E_i^{k-1} \in \{E_1^{k-1}, \dots, E_n^{k-1}\}$;
- 4) $E_i^k = E_i^{k-1} \cup \{e\}$; and delete all edges from e to E_i^k ;
- 5) update $W_e^k(A_S, E_i)$ for all nodes of $G(A_S)$;

- 6) set $k = k + 1$ and go to step (3);
- 7) continue, until there exist no edges.
- 8) $\forall s_1, s_2 \in E^*, s_1 \not\prec s_2, s_2 \not\prec s_1, q, q_1, q_2 \in Q, \delta(q, s_1) = q_1 \neq \delta(q, s_2) = q_2, [\nexists e_1, e_2 \in E, e_1 e_2 \leq s_1, e_2 e_1 \leq s_2, \forall t \in E^*, \delta(q, e_1 e_2 t)! \Leftrightarrow \delta(q, e_2 e_1 t)!], \exists e \in s_1 \cap s_2$, then $\forall i \in loc(e), \forall e' \in \{e_1 \leq t_1, e_2 \leq t_2\}, e'$ appears before e , include e' in E_i .
- 9) go to Step 1 and continue until $\forall s_1, s_2 \in E^*, s_1 \not\prec s_2, s_2 \not\prec s_1, q, q_1, q_2 \in Q, \delta(q, s_1) = q_1 \neq \delta(q, s_2) = q_2, \exists e \in s_1 \cap s_2, [\nexists e_1, e_2 \in E, e_1 e_2 \leq s_1, e_2 e_1 \leq s_2, \forall t \in E^*, \delta(q, e_1 e_2 t)! \Leftrightarrow \delta(q, e_2 e_1 t)!]$, then $\forall i \in loc(e), E_i$ contains the first events of s_1 and s_2 , that appear before e .

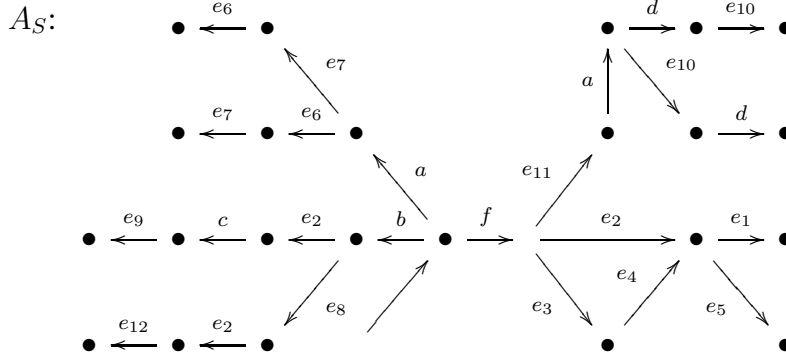
Based on this formulation, a solution to Problem 1 is given as the following theorem.

Theorem 1: Consider a deterministic task automaton A_S with local event sets E_i such that $E = \bigcup_{i=1}^n E_i$. If A_S is not decomposable with respect to parallel composition and natural projections $P_i, i = 1, \dots, n$, Algorithm 3 makes A_S decomposable. Moreover, if after Step 7, $DC3$ and $DC4$ are satisfied, then the algorithm makes A_S decomposable, with minimum number of communication links.

Proof: After excluding the redundant shared events in the first step, the algorithm enforces $DC1$ and $DC2$ in Steps 2 to 7, according to Lemma 4 and deals with $DC3$ and $DC4$ in Steps 8 and 9, based on Lemma 8. ■

Remark 7: If after Step 7, no violation of $DC3$ or $DC4$ is reported in the automaton, then A_S is made decomposable with minimum number of added communication links; otherwise, the optimal solution can be obtained through exhaustive search by examining the number of added links for any possible sequence of enforcing tuples, using Lemmas 5 and 6, as it was presented in Lemma 7. To avoid the exhaustive search the algorithm provides a sufficient condition to enforce $DC3$ and $DC4$ in Steps 8 and 9, according to Lemma 8. The algorithm terminates, due to finite number of states and events, and the fact that at the worst case, when all events are shared among all agents, the task automaton is trivially decomposable.

Example 12: Consider a task automaton



with local event sets $E_1 = \{a, b, c, d, f, e_1, e_3, e_5, e_7, e_9, e_{11}\}$ and $E_2 = \{a, b, c, d, f, e_2, e_4, e_6, e_8, e_{10}, e_{12}\}$, with the communication pattern $2 \in \text{snd}_{a,b,c,d}(1)$ and no more communication links. This task automaton is not decomposable, due to the set of $DC1\&2$ -violating tuples $\{e_1, e_2\}$, $\{e_1, e_4\}$, $\{e_2, e_3\}$, $\{e_2, e_5\}$, $\{e_3, e_4\}$, $\{e_4, e_5\}$, $DC3$ -violating tuples $(e_{11}ade_{10}, ae_7e_6, a, E_1, E_2)$, $(e_{11}ade_{10}, ae_6e_7, a, E_1, E_2)$, $(e_{11}ae_{10}d, ae_7e_6, a, E_1, E_2)$, $(e_{11}ae_{10}d, ae_6e_7, a, E_1, E_2)$ and $DC4$ -violating tuple $(q_0, e_{11}, \varepsilon, a, E_2)$. There is also one event d that is redundantly shared with E_2 as d is passive in E_2 and its exclusion respects $EF1$ - $EF4$. Therefore, at the first step, the algorithm excludes d from E_2 .

Next step is to construct the $DC1\&2$ -Violating graph and remove its edges by sharing one node from each edge. The set of $DC1\&2$ -Violating event pair is obtained as $V^0(A_S) = \{\{e_1, e_2\}, \{e_1, e_4\}, \{e_2, e_3\}, \{e_2, e_5\}, \{e_3, e_4\}, \{e_4, e_5\}\}$ with $W^0(A_S) = \{e_1, e_2, e_3, e_4, e_5\}$. It can be seen that the private events $d, e_6, e_7, e_8, e_9, e_{10}, e_{11}, e_{12}$, and shared events a, b, c, f are not included in $W^0(A_S)$ as they have no contribution in violation of $DC1$ and $DC2$. The $DC1\&2$ -Violating graph is shown in Figure 4(a).

The maximum $|W_e^{k-1}(A_S, E_i^{k-1})|$ is formed by $\{e_2, e_4\}$ with respect to E_1 (here, since the system has only two local event sets $|W_e^{k-1}(A_S, E_i^{k-1})|$ coincides to the valency of e in the graph). Marking e_2 , including it to E_1 ($E_1^1 = \{a, b, c, d, e_1, e_3, e_5, e_7, e_9, e_{11}, e_2\}$) and removing its incident edges to E_1 and updating the $|W_e^k(A_S, E_i^k)|$ (valencies) are shown in Figure 4(b). The next step will include e_4 in E_1 ($E_1^2 = \{a, b, c, d, e_1, e_3, e_5, e_7, e_9, e_{11}, e_2, e_4\}$) with the highest $|W_e^k(A_S, E_i^k)|$ and removing its incident edges to E_1 and updating the $|W_e^k(A_S, E_i^k)|$ will accomplish enforcing of $DC1$ and $DC2$ upon Step 7, as it is illustrated in Figure 4 (c). If from the first stage e_4 was chosen instead of e_2 , the procedure was similarly performed as depicted in Figures 4 (d) and (e), resulting the same set of private events $\{e_2, e_4\}$ to be shared with E_1 . Inclusion of e_2 in E_1 , however, introduces a new $DC4$ -violating tuple $(\delta(q_0, b), \varepsilon, e_8, e_2, E_1)$

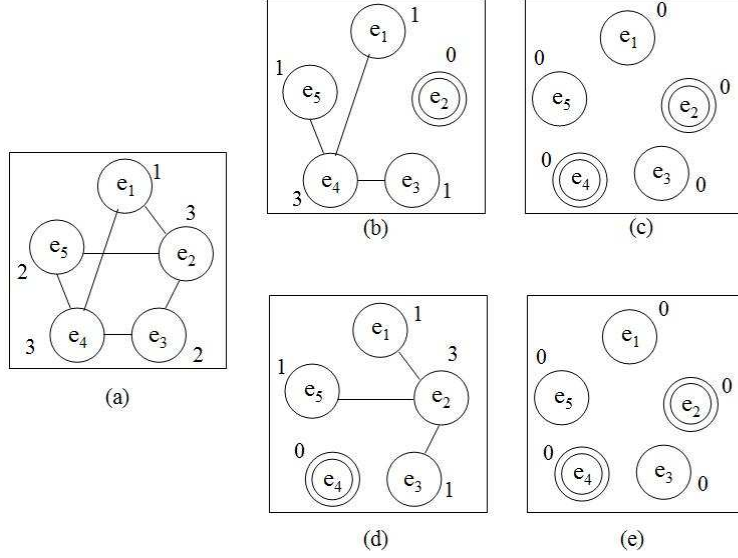


Fig. 4. Illustration of enforcing $DC1$ and $DC2$ in Example 12, using Algorithm 3.

that will be automatically overcome in Step 8 by sharing $e_8 \in s_1 = e_8 e_2 e_{12}$ (as $s_1 = e_8 e_2 e_{12}$ together with $s_2 = e_2 c e_9$ evolve from $\delta(q_0, b)$, sharing $e_2 \in s_1 \cap s_2$) in all local event sets of e_2 , i.e., by including e_8 into E_1 . Similarly, inclusion of e_{11} in E_2 overcomes $DC4$ -violating tuple $(q_0, e_{11}, \varepsilon, a, E_2)$. It is worth noting that the expression “ $\nexists e_1, e_2 \in E, e_1 e_2 \leq s_1, e_2 e_1 \leq s_2, \forall t \in E^*, \delta(q, e_1 e_2 t)! \Leftrightarrow \delta(q, e_2 e_1 t)!$ ” in Step 8 prevents unnecessary inclusion of e_{10} in E_1 as well as e_7 in E_2 and e_6 in E_1 (e_6 and e_7 satisfy $DC1$ - $DC2$ and e_{10} and d satisfy $EF1$ - $EF2$). The algorithm terminates in this stages, leading to decomposability of A_S , with $E_1^3 = \{a, b, c, d, e_1, e_3, e_5, e_7, e_9, e_{11}, e_2, e_4, e_8\}$, $E_2^3 = E_2$, $E_2^3 = \{a, b, c, e_2, e_4, e_6, e_8, e_{10}, e_{11}, e_{12}\}$.

IV. CONCLUSIONS

The paper proposed a method for task automaton decomposabilization, applicable in top-down cooperative control of distributed discrete event systems. This result is a continuation of our previous works on task automaton decomposition [19], [20], and fault-tolerant cooperative tasking [21], and investigates the follow-up question to understand that how an originally undecomposable task automaton can be made decomposable, by modifying the event distribution among the agents.

First, using the decomposability conditions the sources of undecomposability are identified and then a procedure was proposed to establish new communication links in order to enforce

the decomposability conditions. To avoid the exhaustive search and the difficulty of checking of decomposability conditions in each step, a feasible solution was proposed as a sufficient condition that can make any deterministic task automaton decomposable.

V. APPENDIX

A. Proof of Lemma 4

Following lemma will be used during the proof.

Lemma 9: Consider two non-increasing chains $a_i, b_i, i = 1, \dots, N$, such that $a_1 \geq a_2 \geq \dots \geq a_N > 0, b_1 \geq b_2 \geq \dots \geq b_N > 0$. Then $\sum_{i=1}^N a_i < \sum_{i=1}^N b_i$ implies that $\exists k \in \{1, \dots, N\}$ such that $a_k < b_k$.

Proof: Suppose by contradiction that $\sum_{i=1}^N a_i < \sum_{i=1}^N b_i$, but, $\nexists k \in \{1, \dots, N\}$ such that $a_k < b_k$. Then, $\forall k \in \{1, \dots, N\} : a_k \geq b_k$. Therefore, since $a_k, b_k > 0, \forall k \in \{1, \dots, N\}$, it results in $\sum_{i=1}^N a_i \geq \sum_{i=1}^N b_i$ which contradicts to the hypothesis, and the proof is followed. ■

Now, we prove Lemma 4 as follows. In each iteration k for the event e and local event set E_i with maximum $|W_e^{k-1}(A_S, E_i^{k-1})|$, all edges from e to E_i are deleted. Denoting the set of deleted edges in k -th iterations by $\Delta\Sigma^k$, in each iteration k , some elements of Σ^{k-1} are moved into $\Delta\Sigma^k$ until after K iterations, there is no more elements in Σ^K to be moved into a new set. This iterative procedure leads to a partitioning of Σ by $\{\Delta\Sigma^k\}_{k=1}^K$, as $\{\Delta\Sigma^k\} \cap \{\Delta\Sigma^l\} = \emptyset, \forall k, l = \{1, \dots, K\}, k \neq l$ and $\bigcup_{k=1}^K \Delta\Sigma^k = \Sigma$. The latter equality leads to

$$\sum_{k=1}^K |\Delta\Sigma^k| = |\Sigma| \quad (1)$$

Now, we want to prove that

$$|\Delta\Sigma^k| = |\Delta\Sigma^k|_{\max}, \forall k \in \{1, \dots, K\} \Rightarrow K = K_{\min} \quad (2)$$

Here, K is the total number of iterations that is also equal to the number of added communication links to remove violations of *DC1* and *DC2*. In this sense, K is desired to be minimized.

The proof of (2) is by contradiction as follows. Suppose that $|\Delta\Sigma^k| = |\Delta\Sigma^k|_{\max}, \forall k \in \{1, \dots, K\}$, but, $K \neq K_{\min}$, i.e., there exists another partitioning $\{\Delta'\Sigma^k\}_{k=1}^{K'}$, with $K' < K$

partitions, leading to

$$\sum_{k=1}^{K'} |\Delta' \Sigma^k| = |\Sigma| \quad (3)$$

In this case, from (1) and (3), we have

$$\sum_{k=1}^K |\Delta \Sigma^k| = \sum_{k=1}^{K'} |\Delta \Sigma^k| + \sum_{k=K'+1}^K |\Delta \Sigma^k| = \sum_{k=1}^{K'} |\Delta' \Sigma^k|. \quad (4)$$

Since $|\Delta \Sigma^k| > 0, \forall k \in \{1, \dots, K\}$, then $\sum_{k=K'+1}^K |\Delta \Sigma^k| > 0$, then, (4) results in

$$\sum_{k=1}^{K'} |\Delta \Sigma^k| < \sum_{k=1}^{K'} |\Delta' \Sigma^k|. \quad (5)$$

Moreover, since $|\Delta \Sigma^k| > 0, |\Delta' \Sigma^k| > 0, \forall k \in \{1, \dots, K\}$, then (5) together with Lemma 9 imply that $\exists k \in \{1, \dots, K'\} \subseteq \{1, \dots, K\}$, i.e., $|\Delta \Sigma^k| < |\Delta' \Sigma^k|$, i.e., $\exists k \in \{1, \dots, K\}$ such that $|\Delta \Sigma^k| \neq |\Delta \Sigma^k|_{max}$, which contradicts to the hypothesis, and hence, (2) is proven. Moreover, if automaton A_S has no violations of *DC3* and *DC4* before and during the iterations, then the algorithm make it decomposable with the minimum number of added communication links, since the problem of making decomposable is reduced to optimal enforcing of *DC1* and *DC2*.

B. Proof for Lemma 5

For any *DC3*-violating tuple (s_1, s_2, a, E_i, E_j) , exclusion of a from E_i or E_j , excludes a from $E_i \cap E_j$, leading to $p_{E_i \cap E_j}(s_1)$ and $p_{E_i \cap E_j}(s_2)$ do not start with a , and hence (s_1, s_2, a, E_i, E_j) will no longer act as a *DC3*-violating tuple.

For the second method in this lemma, firstly $\forall q \in Q, s_1, s_2 \in E^*, \delta(q, s_1)!, \delta(q, s_2)!, p_{E_i \cap E_j}(s_1)$ and $p_{E_i \cap E_j}(s_2)$ start with a , such that (s_1, s_2, a, E_i, E_j) is a *DC3*-violating tuple, $\exists b \in (E_i \cup E_j) \setminus (E_i \cap E_j)$ such that b appears before a in s_1 or s_2 (since A_S is deterministic and $p_{E_i \cap E_j}(s_1)$ and $p_{E_i \cap E_j}(s_2)$ start with a).

Two cases are possible, here: b appears in only one of the strings s_1 or s_2 ; or b appears in both strings. If b appears before a in only of the strings, then without loss of generality, assume that b belongs to only s_1 , and hence, $\exists q, q_1, q_2, q'_1, q''_1 \in Q_i \times Q_j, \omega_1, \omega_2 \in [(E_i \cup E_j) \setminus (E_i \cap E_j)]^*, \omega'_1 \in (E_i \cup E_j)^*, a \in E_i \cap E_j$ such that $\delta_{i,j}(q, \omega_1) = q'_1, \delta_{i,j}(q'_1, b) = q''_1, \delta_{i,j}(q''_1, \omega'_1) = q_1, \delta_{i,j}(q_1, a)!, \delta_{i,j}(q, \omega_2) = q_2, \delta_{i,j}(q_2, a)!$, where, $\delta_{i,j}$ is the transition relation in $P_i(A_S) || P_j(A_S)$. Now, due to synchronization constraint in parallel composition, inclusion of b in $E_i \cap E_j$ means

that $([q_1''], y)$ and $(x, [q_1'']_j)$ are accessible in $P_i(A_S) || P_j(A_S)$ only if $y = [q_1'']_j$ and $x = [q_1'']_i$, respectively. This means that $([q_1]_i, [q_2]_j)$ and $([q_2]_i, [q_1]_j)$ are not accessible in $P_i(A_S) || P_j(A_S)$, and hence, $p_i(s_1) | p_j(s_2)$ and $p_i(s_2) | p_j(s_1)$ cannot evolve after a , and therefore, do not generate illegal strings out of the original strings, implying that (s_1, s_2, a, E_i, E_j) will no longer remain a *DC3*-violating tuple.

On the other hand, if b appears before a , in both strings s_1 and s_2 , then $\exists q, q_1, q_2, q'_1, q''_1, q'_2, q''_2 \in Q_i \times Q_j, \omega_1, \omega_2 \in [(E_i \cup E_j) \setminus (E_i \cap E_j)]^*, \omega'_1, \omega'_2 \in (E_i \cup E_j)^*, a \in E_i \cap E_j$ such that $\delta_{i,j}(q, \omega_1) = q'_1, \delta_{i,j}(q'_1, b) = q''_1, \delta_{i,j}(q''_1, \omega'_1) = q_1, \delta_{i,j}(q_1, a)! , \delta_{i,j}(q, \omega_2) = q'_2, \delta_{i,j}(q'_2, b) = q''_2, \delta_{i,j}(q''_2, \omega'_2) = q_2, \delta_{i,j}(q_2, a)!$, that leads to accessibility of $([q'_1]_i, [q'_2]_j)$ and $([q'_2]_i, [q'_1]_j)$ as well as $([q_1]_i, [q_2]_j)$ and $([q_2]_i, [q_1]_j)$ in $P_i(A_S) || P_j(A_S)$, that means that although (s_1, s_2, a, E_i, E_j) is no longer a *DC3*-violating tuple, (s_1, s_2, b, E_i, E_j) emerges as a new *DC3*-violating tuple.

In this case (when $\nexists b \in (E_i \cup E_j) \setminus (E_i \cap E_j)$ that appears before a in only one of the strings s_1 or s_2), instead of inclusion of b in $E_i \cap E_j$, if two different events that appear before a in strings $p_{E_i \cup E_j}(s_1)$ and $p_{E_i \cup E_j}(s_1)$ are attached to $E_i \cap E_j$, it leads to $\exists q, q_1, q_2, q_3, q_4 \in Q_i \times Q_j, \omega_1, \omega_2, \omega'_1, \omega'_2 \in [(E_i \cup E_j) \setminus (E_i \cap E_j)]^*, e_1, e_2, a \in E_i \cap E_j$ such that $\delta_{i,j}(q, \omega_1 e_1) = q_1, \delta_{i,j}(q_1, \omega'_1) = q_3, \delta_{i,j}(q_3, a)! , \delta_{i,j}(q, \omega_2 e_2) = q_2, \delta_{i,j}(q_2, \omega'_2) = q_4, \delta_{i,j}(q_4, a)!$. Consequently, due to synchronization constraint in parallel composition, $([q_1]_i, [q]_j), ([q]_i, [q_1]_j), ([q_2]_i, [q]_j)$ and $([q]_i, [q_2]_j)$, and hence, $([q_3]_i, [q_4]_j)$ and $([q_4]_i, [q_3]_j)$ are not accessible in $P_i(A_S) || P_j(A_S)$, i.e., no more *DC3*-violating tuples form on strings s_1 and s_2 .

C. Proof for Lemma 6

For any *DC4*-violating tuple (q, t_1, t_2, e, E_i) , with $q, q_1, q_2 \in Q, t_1, t_2 \in (E \setminus E_i)^*, e \in E_i, \delta(q, t_1) = q_1 \neq \delta(q, t_2) = q_2$, exclusion of e from E_i leads to $p_i(e) = \varepsilon$, and $p_i(t_1 e) = p_i(t_2 e) = \varepsilon, [q]_i = [\delta(q_1, e)]_i = [\delta(q_2, e)]_i$, and hence, (q, t_1, t_2, e, E_i) will no longer behave as a *DC4*-violating tuple. However, it should be noted that it may cause another nondeterminism on an event after e , and this event exclusion is allowed only if e is passive in E_i and the exclusion does not violate *EF1* – *EF4*.

For the second method, i.e., event inclusion, if $\exists e' \in (t_1 \cup t_2) \setminus (t_1 \cap t_2)$, then without loss of generality, assume that $e' \in t_1 \setminus t_2$ such that $\exists q, q_1, q_2, q'_1, q''_1 \in Q, t_1, t_2 \in (E \setminus E_i)^*, e \in E_i, \delta(q, t_1) = q_1 \neq \delta(q, t_2) = q_2, \delta(q'_1, e') = q''_1$. In this case, inclusion of e' in E_i leads to $p_i(t_1 e) = e' e$, while $p_i(t_2 e) = e$, and therefore, $[q_1]_i = [q''_1]_i \neq [q_2]_i$, i.e., in $P_i(A_S)$, t_1 and t_2 will no

longer cause a nondeterminism on e from q , and accordingly, (q, t_1, t_2, e, E_i) will not remain a $DC4$ -violating tuple.

If however $\nexists e' \in (t_1 \cup t_2) \setminus (t_1 \cap t_2)$, i.e., $\forall e' \in (t_1 \cup t_2), e' \in (t_1 \cap t_2)$, then inclusion of any such e' generates a $DC4$ -violating tuple (q, t_1, t_2, e', E_i) . In this case, Lemma 6 suggests to take two different events that appear before e , one from t_1 and the other from t_2 , and include them into E_i such that $\exists q, q_1, q_2, q'_1, q'_2, q''_1, q''_2 \in Q, e_1 \in t_1, e_2 \in t_2, e_1 \neq e_2, \delta(q, t_1) = q_1 \neq \delta(q, t_2) = q_2, \delta(q'_1, e_1) = q''_1, \delta(q'_2, e_2) = q''_2$. Thus, including e_1 and e_2 in E_i results in $p_i(t_1) = e_1, p_i(t_2) = e_2, \delta_i([q]_i, t_1)! = [q_1]_i \neq \delta_i([q]_i, t_2) = [q_2]_i$, meaning that (q, t_1, t_2, e, E_i) is not a $DC4$ -violating tuple anymore.

D. Proof for Lemma 7

The algorithm starts with excluding events from local event sets in which the events are passive and their exclusion do not violate $EF1$ - $EF4$. From this stage onwards the decomposability conditions are no longer allowed to be enforced by link deletion, whereas the algorithm removes the violations of decomposability conditions by establishing new communication links. Next, the algorithm applies violating tuples in the order of corresponding number of violating tuples. If no new violations of decomposability conditions emerge during conducting of enforcing tuples, then the algorithm decomposes the task automaton with minimum number of communication links, similar to the proof of Lemma 4, since iterations partition the set of violating tuples, and applying of enforcing tuples (based on Lemmas 4, 5 and 6) with maximum number of violating tuples in each iteration gives maximum number of resolutions per link addition that leads to the minimum number of added communication links. The algorithm will terminate due to finite number of states and events and at the worst case all events are shared among all agents to make the automaton decomposable.

E. Proof for Lemma 8

Denoting the expression , “ $\forall E_i, E_j \in \{E_1, \dots, E_n\}, i \neq j, a \in E_i \cap E_j, s = t_1 a t'_1, s' = t_2 a t'_2, p_{E_i \cap E_j}(t_1) = p_{E_i \cap E_j}(t_2) = \varepsilon$ ” as A , and the expression “ $\delta(q_0, \bigcup_{i=1}^n p_i(s_i))!$ for any $\forall \{s_1, \dots, s_n\} \subseteq \tilde{L}(A_S), s, s' \in \{s_1, \dots, s_n\}$ ” as B , the condition $DC3$ can be written as $A \Rightarrow B$. Now, if $\forall s_1, s_2 \in E^*, s_1 \not\leq s_2, s_2 \not\leq s_1, q, q_1, q_2 \in Q, \delta(q, s_1) = q_1 \neq \delta(q, s_2) = q_2, [\nexists e_1, e_2 \in E, e_1 e_2 \leq s_1, e_2 e_1 \leq s_2, \forall t \in E^*, \delta(q, e_1 e_2 t)! \Leftrightarrow \delta(q, e_2 e_1 t)!], \exists e \in s_1 \cap s_2, \text{ any } e' \in \{e_1 \leq t_1, e_2 \leq t_2\}, \text{ such}$

that e' appears before e , is included in E_i , $\forall i \in \text{loc}(e)$, it follows that $\forall E_i, E_j \in \{E_1, \dots, E_n\}$, $i \neq j$, $a \in E_i \cap E_j$, $s = t_1 a t'_1$, $s' = t_2 a t'_2$, $\delta(q_0, s)! \neq \delta(q_0, s')!$, $a \in s \cap s'$, then the first event of t_1 and t_2 belong to $E_i \cap E_j$, i.e., A (the antecedent of $DC3$) becomes false, and hence, $A \Rightarrow B$ ($DC3$) holds true. Therefore, the procedure in Lemma 8 gives a sufficient conditions to make $DC3$ always true.

It is similarly a sufficient condition for $DC4$ as follows. Let the expressions “ $\forall i \in \{1, \dots, n\}$, $x, x_1, x_2 \in Q_i, e \in E_i, t \in E_i^*, \delta(x, e) = x_1 \neq \delta(x, e) = x_2$ ” and “ $\forall t \in E_i^* : \delta(x_1, t)! \Leftrightarrow \delta(x_2, t)!$ ” to be denoted as C and D , respectively. In this case, $DC4$ can be expressed as $C \Rightarrow D$. Then, for a deterministic automaton A_S , if $\forall s_1, s_2 \in E^*$, $s_1 \not\leq s_2$, $s_2 \not\leq s_1$, $q, q_1, q_2 \in Q$, $\delta(q, s_1) = q_1 \neq \delta(q, s_2) = q_2$, $[\nexists e_1, e_2 \in E, e_1 e_2 \leq s_1, e_2 e_1 \leq s_2, \forall t \in E^*, \delta(q, e_1 e_2 t)! \Leftrightarrow \delta(q, e_2 e_1 t)!]$, $\exists e \in s_1 \cap s_2$, the first event of s_1 and s_2 are included in all local event sets that contain e , it results in $\neg C$ (i.e., the antecedent of $DC4$ becomes false, and consequently, $DC4$ becomes always true), since in such case $\forall E_i \in \{E_1, \dots, E_n\}$, $t_1, t_2 \in E^*$, $q, q_1, q_2 \in Q$, $e \in E_i$, $\delta(q, t_1 e) = q_1 \neq \delta(q, t_2 e) = q_2$, then $\neg [p_i(t_1) = p_i(t_2) = \varepsilon]$.

Expression “ $[\nexists e_1, e_2 \in E, e_1 e_2 \leq s_1, e_2 e_1 \leq s_2, \forall t \in E^*, \delta(q, e_1 e_2 t)! \Leftrightarrow \delta(q, e_2 e_1 t)!]$ ”, in Lemma 8 is to exclude those pairs of strings s_1 and s_2 that start with $e_1 e_2$ and $e_2 e_1$, respectively, as they have been already checked with $DC1$ and $DC2$ and their interleaving does not impose illegal strings.

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